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Critical point theory of earthquakes: Observation of correlated and cooperative behavior on earthquake fault systems

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[1] The critical point theory for earthquakes was originally proposed to explain the scaling relations observed in earthquakes, including the Gutenberg-Richter frequency-magnitude relation and the Omori's law for aftershocks. In this model, main shocks, their foreshocks and aftershocks are all associated with the formation of a correlated, cooperative spatial region with high stress. Until now, only indirect evidence of the existence of these correlated regions has been reported. Here in this paper we present observations and analyses that allow us to directly map the high stress, spatially correlated regions preceding four major earthquakes, i.e. the 1992 Landers (California), 1995 Kobe (Japan), 1999 Chi-Chi (Taiwan) and 1999 Hector Mine (California) earthquakes. We therefore conclude that the locations and extent of large main shocks and their immediate aftershocks can be determined from seismicity data taken prior to the main shocks, and provide additional evidence in support of the critical point theory for earthquakes. **Citation:** Chen, C., J. B. Rundle, H.-C. Li, J. R. Holliday, D. L. Turcotte, and K. F. Tiampo (2006), Critical point theory of earthquakes: Observation of correlated and cooperative behavior on earthquake fault systems, *Geophys. Res. Lett.*, *33*, L18302, doi:10.1029/2006GL027323.

1. Introduction to the Critical Point Theory of Earthquakes

[2] The critical point theory for earthquakes has been proposed to understand important empirical scaling laws in seismology [*Allegre et al.*, 1982; *Sornette and Sornette*, 1990; *Bak*, 1996; *Jaume and Sykes*, 1999; *Zöller et al.*, 2001; *Rundle et al.*, 2003]. The scaling properties of earthquake populations, for instance, show remarkable similarities to those observed among the critical phenomena of magnetic or other composite systems in statistical physics [*Ma*, 1976; *Bak*, 1996]. In the last two decades many researchers have thus attempted to model the earthquake process by analogy with the statistical mechanics of critical phenomena, culminating in a great earthquake that is analogous to a kind of critical point [*Sornette and Sornette*, 1990; *Rundle et al.*, 1997, 1999, 2003; *Jaume and Sykes*, 1999; *Zöller et al.*, 2001]. Evidence cited for considering earthquakes to be an example of critical phenomena is the fact that great earthquakes are frequently observed to be preceded by accelerating seismicity occurring over a region much larger than the rupture zones [*Sykes and Jaume*, 1990; *Bowman et al.*, 1998; *Jaume and Sykes*, 1999; *Zöller et al.*, 2001; *Chen*, 2003].

[3] Fluctuations associated with correlations in space and time are inherent in critical phenomena [*Stanley*, 1971; *Ma*, 1976]. Laboratory observations demonstrate the existence of very large spatial and temporal fluctuations in the density of the liquid-vapor mixture near the critical point, and in the magnetization of ferromagnets near the Curie point [*Stanley*, 1971; *Ma*, 1976]. Observations also indicate that these fluctuations are correlated over distance and time scales that can be characterized by correlation lengths and the correlation times, respectively. Since earthquakes represent a release of accumulated stress, the critical point theory of earthquakes would therefore predict that main shock and its aftershocks would be associated with a strongly correlated spatial region of high stress that forms prior to the main shock. Indirect evidence for the existence of such correlated regions of high stress has been reported, for example in the appearance of time-dependent variations in the form of the Gutenberg-Richter relation [*Jaume and Sykes*, 1999; *Chen*, 2003], and in the apparent correlation in seismic activity over large distances [*Bowman et al.*, 1998; *Zöller et al.*, 2001]. The time-dependent variations in the frequency-magnitude statistics indicate changes in correlation length of earthquake fault systems and can be described in the context of the self-organizing spinodal (SOS) model of earthquakes proposed by *Rundle et al.* [2000b]. The SOS model is a description of a first-order phase transition and, for such phase transition, proximity to the spinodal leads to the observation of scaling. The spinodal behaves exactly like a line of critical points [*Klein et al.*, 1997; *Rundle et al.*, 2000b].

[4] In this paper we apply the method of pattern informatics (PI) to map high stress, correlated regions [*Rundle et al.*, 2000a, 2002, 2003; *Tiampo et al.*, 2002a, 2002b, 2002c; *Chen et al.*, 2005] with rapid changes in seismicity for smaller events associated with four major earthquakes: the 28 June 1992 Landers (California), 17 January 1995 Kobe (Japan), 21 September 1999 Chi-Chi (Taiwan) and 16 October 1999 Hector Mine (California) earthquakes. We have chosen these four major earthquakes because of the richness of the aftershock sequences. The PI technique reveals regions of strongly correlated fluctuations in the background seismic activity. These regions are the locations where subsequent large earthquakes, including both main shock and aftershock sequence, have been shown to occur,

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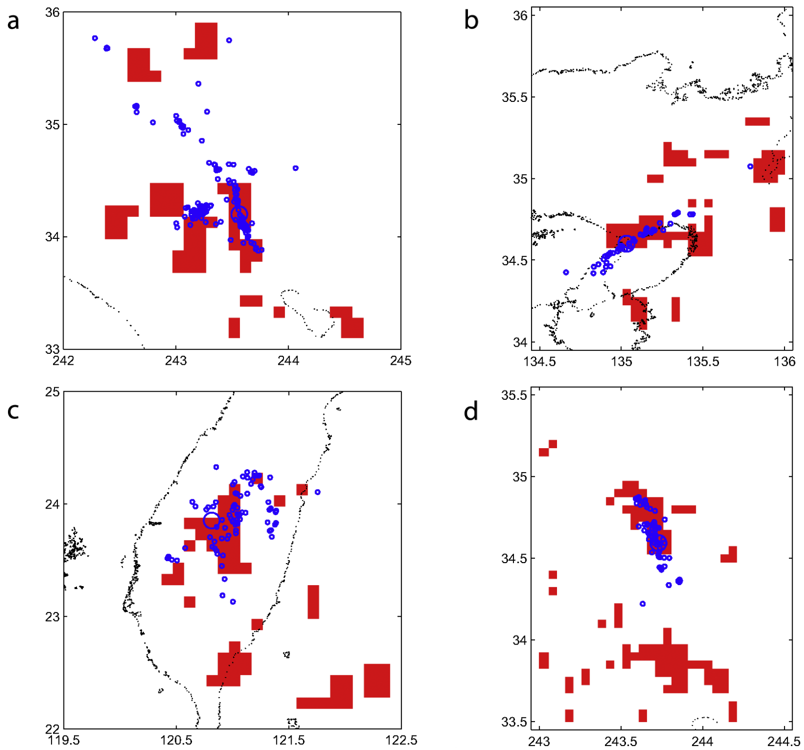

Figure 1. PI maps around the (a) $M = 7.2$ Landers, (b) $M = 7.2$ Kobe, (c) $M = 7.3$ Chi-Chi and (d) $M = 7.1$ Hector Mine earthquakes. These maps were derived from the PI calculation of the background seismicity before main shock. The grid sizes used in PI calculation are $0.1° \times 0.1°$ in Figures 1a and 1c and $0.05° \times 0.05°$ in Figures 1b and 1d. Red pixels represent the areas with large PI indices. Blue circles denote the epicenters of main shocks (large ones) and larger aftershocks occurred in the first 3 months after main shock.

therefore indicating a strong association with the high stress regions that formed prior to major earthquakes. We thus by means of the PI method can observe some kind of spatial correlation associated with major earthquakes and their immediate aftershocks emerging from the fluctuation in background seismicity.

2. Brief Review to the PI Method

[5] PI is based on the idea that the time evolution of seismicity can be described by the rotation of a state vector in a phase dynamical system [*Mori and Kuramoto*, 1997]. The rotations of a state vector in a Hilbert space are interpreted in terms of stress accumulation and release [*Rundle et al.*, 2002, 2003]. The strong space-time correlations in the driven threshold systems, such as earthquake fault systems, are responsible for the cooperative behavior in the system [*Rundle et al.*, 2000a; *Tiampo et al.*, 2002a]. The space-time pattern of threshold earthquakes is represented by a time-dependent state vector in a high-dimensional Hilbert space [*Tiampo et al.*, 2002a, 2002b, 2002c]. State vectors are defined in the space of eigenvectors obtained from a seismicity correlation matrix [*Rundle et al.*, 2000a; *Tiampo et al.*, 2002a]. Since the norm ("length") of the state vector is constant, information about the space-time fluctuations in the system is carried only by the phase angle of the state vector. Temporal variation in seismicity corresponds to a time-dependent drift in the phase angle over a defined period of time, the "change interval" [*Chen et al.*, 2005]. The drift points towards the direction of future seismicity patterns in the Hilbert space, allowing regions of high correlation in activity and stress to be located and mapped [*Rundle et al.*, 2000a, 2003; *Tiampo et al.*, 2002a]. Practically the PI calculation estimates a mean drift angle, i.e. the vector difference, over the change interval and the probability amplitudes of threshold events in the future can be derived from the mean drift angle. The result is the PI index, which is the average squared change in seismic intensity over the change interval, for each spatial grid box or pixel. For many details in the PI calculation, please refer to a very recently published paper by *Chen et al.* [2005, and references therein].

3. Correlated PI Patterns Between Main Shocks and Their Immediate Aftershocks

[6] Figure 1 shows the results of our retrospective analyses, four PI maps around the epicenters of the (a) $M = 7.2$ Landers, (b) $M = 7.2$ Kobe, (c) $M = 7.3$ Chi-Chi and (d) $M = 7.1$ Hector Mine earthquakes, respectively. Red pixels represent the areas with high PI indices above some given threshold and indicate the potential locations of future events. The change intervals used in our PI calculation for these cases are (a) from July 1986 through March 1992, (b) from January 1989 through December 1994, (c) from November 1993 through June 1999 and (d) from April 1997 through August 1999. Such choices for these change intervals were suggested by the temporal evolution of frequency-magnitude statistics of earthquakes in the context

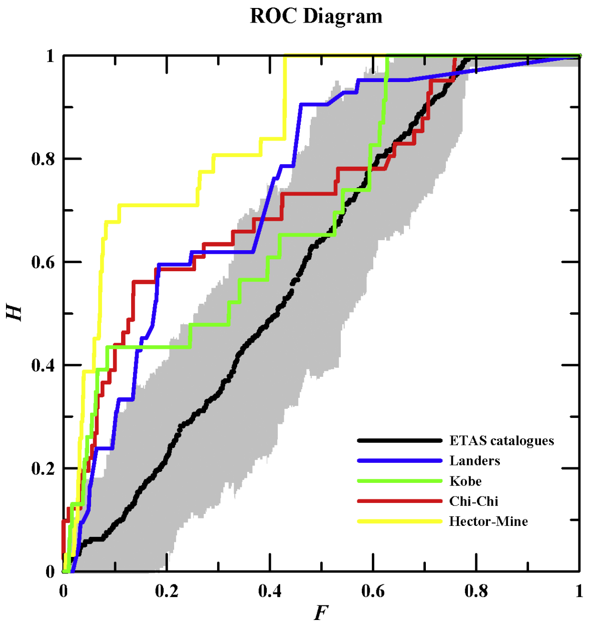

Figure 2. The ROC curves for the ETAS and real catalogues. H and F represent the hit rate and false alarm rate, respectively.

of the SOS model of earthquake fault systems [*Rundle et al.*, 2000b, 2003; *Chen*, 2003; *Chen et al.*, 2005].

[7] For all four PI maps shown in Figure 1, red pixels around the epicenters reveal some correlated patches with high PI indices. An immediate fact appearing on each PI map is that the main shock (large blue circle on each PI map) exactly occurred on the major cluster of connected red pixels, thus strongly suggesting that the epicenter of great earthquake may be associated with an area having severe fluctuations in the background seismicity rate. The typical maximum cluster size associated with the main shock is on the order of 100 km or so, which is consistent with the maximum influence range of 100 $\sim$ 150 km for stacked triggered seismicity as found by *Huc and Main* [2003] and is, particularly for the Kobe earthquake, accordant with the size of the seismic activation region obtained by the Region-Time-Length algorithm of seismicity analysis [*Huang et al.*, 2001]. The fluctuations revealed on a PI map must therefore be related to the preparation process for large earthquake, corresponding to formation of a correlated region with high stress [*Tiampo et al.*, 2002a, 2002b, 2002c]. Seismic quiescence and activation [*Wyss and Habermann*, 1988; *Sykes and Jaume*, 1990; *Wiemer and Wyss*, 1994; *Bowman et al.*, 1998; *Jaume and Sykes*, 1999; *Rundle et al.*, 2000b; *Chen*, 2003; *Huang*, 2004] are examples of such preparation process. The small blue circles in Figure 1 denote the epicenters of larger aftershocks occurring in the first 3 months after main shock, with magnitude above 4 for the Landers,

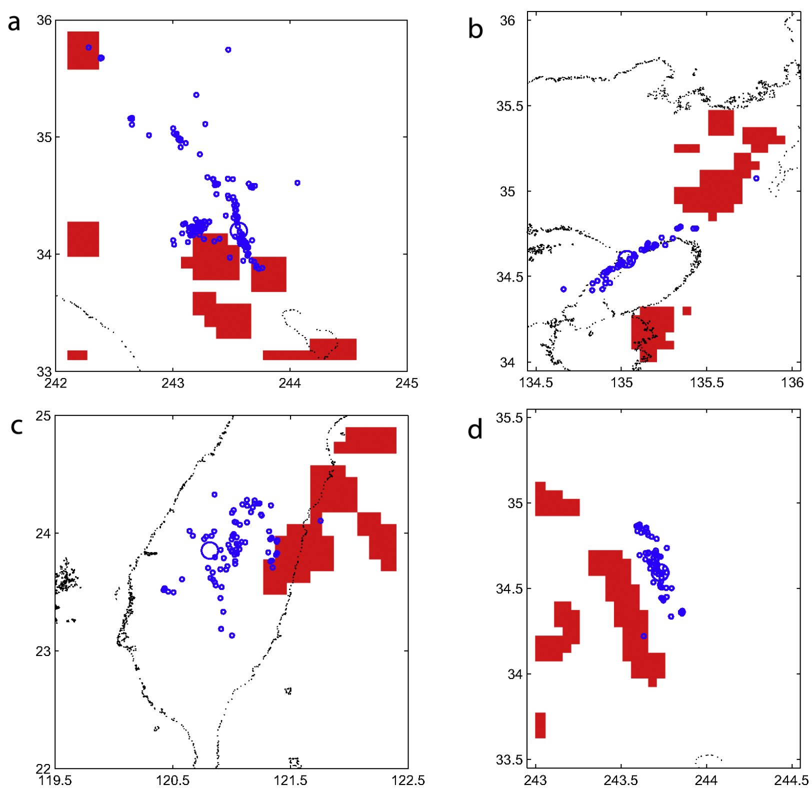

Figure 3. (a–d) RI maps for the same areas corresponding to Figure 1. The earthquakes for making these four RI maps were selected with the same space-time-magnitude windows as selection for doing corresponding PI maps. The grid sizes used are 0.1° × 0.1° in Figures 3a and 3c and 0.05° × 0.05° in Figures 3b and 3d. Red pixels represent the areas with large RI indices. The equal numbers of red pixels are used on both RI and corresponding PI maps (Figure 1). Blue circles denote the epicenters of main shocks (large ones) and larger aftershocks occurred in the first 3 months after main shock.

Kobe and Hector Mine, and 5 for Chi-Chi cases. It is striking that most of the larger aftershocks also occurred on the same major cluster of correlated red pixels on each PI map. Particularly the border of the major cluster appears to be almost coincident with the distributed shape of larger aftershocks.

[8] For demonstrating the statistical significance of observed correlation patterns between main shocks and aftershocks, we have synthesized 20 ETAS (Epidemic Type Aftershock Sequence) catalogues [*Ogata and Zhuang*, 2006] and applied the PI and ROC (Relative Operating Characteristic) analyses to them. As a forecasting verification tool, the ROC analysis was widely adopted [*Chen et al.*, 2005]. For the details of the ROC analysis we refer the reader to the paper of *Chen et al.* [2005]. As shown in Figure 2, the black line represents the mean of ROC curves from 20 ETAS catalogues and the gray envelope the 95% confidence. The ROC curves for four cases of the Landers, Kobe, Chi-Chi and Hector-Mine sequences are also displayed in Figure 2. With the 95% confidence level, the ROC diagram clearly demonstrates that there are differences between the ETAS catalogues and the real earthquake catalogues. Given a F value of 0.2, which indicates a coverage of $\sim$20% over the studied areas, the percentage of events (the Hs) that can be predicted by PI are $\sim$60% and $\sim$25% for the real and ETAS catalogues, respectively. Obviously, the percentages of earthquakes in those simulations of the ETAS null hypothesis are predicted by the PI index significantly less well than the hit percentages in real cases.

[9] We show four relative intensity (RI) maps in Figure 3 with the same space-time-magnitude windows of data selection used for corresponding PI maps. Red pixels on each RI map show the areas having high seismic activities within the given selection window. The RI index for each box is computed as the total number of historic earthquakes occurring in the pixel divided by the value for the pixel having the maximum value. We have plotted the RI maps with the same numbers of red pixels as corresponding PI maps. No signature of the impending major earthquake appears on each RI map. While the RI index is simply a count of the historic earthquake numbers that occurred in a set of spatial grid points, the PI index is derived by considering the fluctuations in many pairs of the RI maps. We thus conclude that the preparation process for these major earthquakes is associated with a change (correlated fluctuation) in the average seismicity rate rather than with the high seismicity rate itself, although both the fluctuation and the mean in seismicity rate were seen to increase prior to the main dynamic stress drop in the laboratory experiments of acoustic emission [*Meredith et al.*, 1990].

[10] We recall that the physical basis of the PI approach is that earthquakes are the result of the self-organizing cooperative behavior and strong space-time correlations arising in an interacting, driven threshold system of faults [*Rundle et al.*, 2000a; *Tiampo et al.*, 2002a]. Both the locations of main shocks and their immediate aftershocks can be well defined by the PI method, leading to the conclusion that the main shock and aftershocks represent the correlated and cooperative behavior from the foreshocks, i.e. the seismicity before main shock, in the earthquake fault system. Without de-clustering aftershock sequence in the PI analysis is a direct manifestation of considering the cooperative behavior in earthquake fault system. Successfully locating both main shock and large aftershocks then strongly implies the space-time correlations may be relevant in earthquake fault system.

4. Implication to the Generation of Aftershocks: Coulomb Stress Transfer or Cooperative Behavior?

[11] Large shallow earthquakes are always followed by aftershocks. Are aftershock locations determined by the stress changes induced by main shock? It is a conventional wisdom in seismology that the main shock rebuilds the crustal stress patterns and aftershock occurring rate climbs where the Coulomb stress increases [*Stein*, 1999; *Toda et al.*, 2002; *McCloskey et al.*, 2005]. However, based on our retrospective analyses of four large earthquakes shown in this study, both the locations of main shock and large aftershocks may have been already determined in advance of the occurrence of main shock. Such a standpoint can be supported by a very recent study [*Schorlemmer and Wiemer*, 2005] on close coincidence between the aftershock locations of the 2004 Parkfield earthquake and the areas with low b values derived from the background seismicity. There is also growing evidence from systematic studies suggesting that the directional dependence of triggering for earthquake pairs does not have the behavior expected from Coulomb triggering [e.g., *Parsons*, 2002; *Huc and Main*, 2003]. Causally speaking, the triggering mechanism of the Coulomb stress changes for aftershocks would not be necessary.

[12] Correlations in space and time play a fundamental role in earthquake process. What we have observed in this work is a spatial correlation between the main shock and its aftershocks, which is formed before the main shock occurrence, since we have derived such correlation pattern from the fluctuation in seismicity occurring prior to the main shock. In the critical point theory of earthquakes, the formation of a correlated region of high stress on an interacting system of earthquake faults is a necessary prerequisite to the occurrence of large threshold event. Our PI method has revealed the existence of a correlated region of seismicity in observational data that precedes the main shock by months or even longer. The fact that this correlated region also correctly locates the aftershocks leads us to identify this region of correlated seismicity with the region of correlated high stress predicted by the theory. We are thus led to a point of view in which we might abandon the usual idea of foreshock, main shock and aftershock [*Bak*, 1996; *Rundle et al.*, 1997, 1999, 2000b, 2003; *Jaume and Sykes*, 1999; *Helmstetter*, 2003] in favor of the view that the earthquake process represents a process of self-organization and cooperative behavior of the entire system. In this view, identifying fluctuations in the space-time patterns of the system will be crucial for predicting the future evolution and activity of the entire system.

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