Probabilistic analysis and seismic hazard assessment with Excel applications

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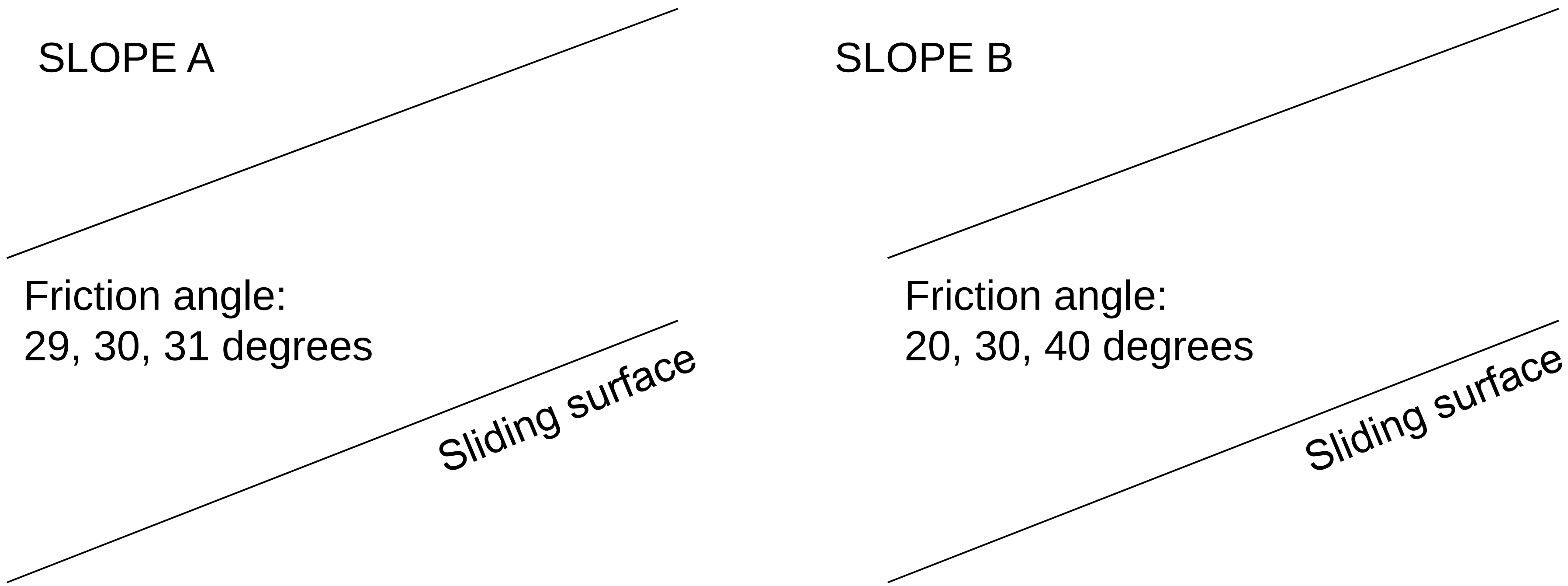
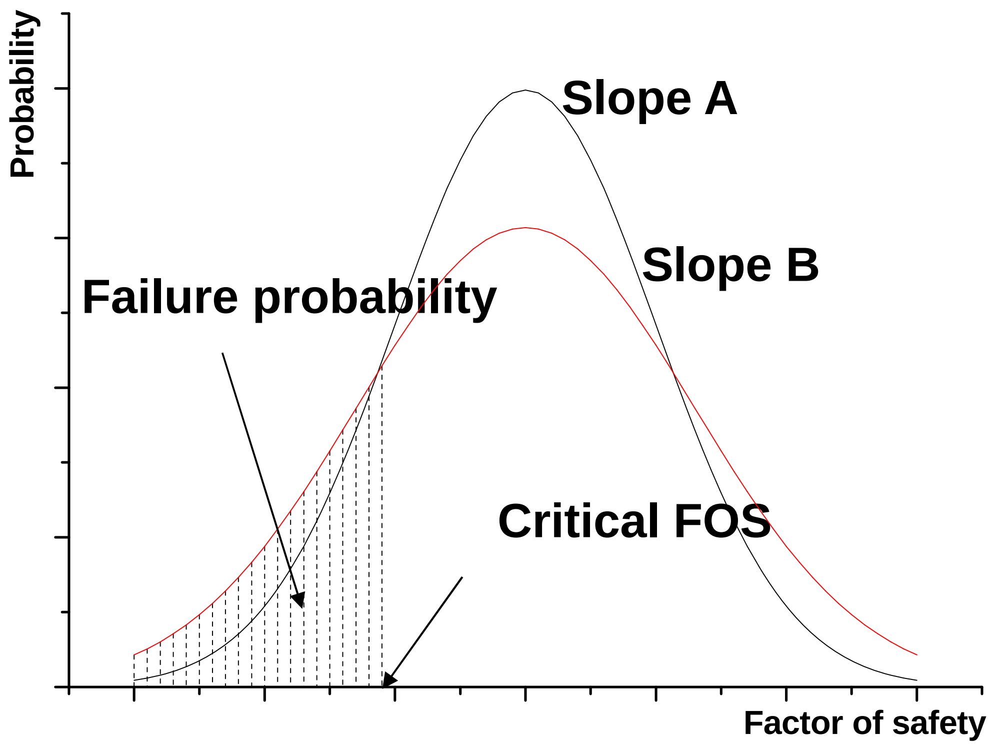
Deterministic analysis



=> Given the same sample's mean, the two slopes are associated with the same factor of safety

Probabilistic analysis

- Probabilistic analysis accounts for the sample's variability as well, estimating the FOS's distribution



Methods for probabilistic analyses

- Monte Carlo Simulation
- First-order-second-moment
- Point-estimate method or Rosenblueth's method
- Etc…

=> One thing in common is the tedious computation involved during analysis

Monte Carlo Simulation (MCS)

- What is the probability getting “1” when tossing a dice?

- Below is the result of tossing the dice 100 times:

1	1	2	1	6	1	2	4	3	5
5	6	2	4	5	5	1	3	4	2
1	5	5	6	3	3	5	6	3	5
5	5	6	1	6	2	6	3	5	1
3	4	4	1	3	1	5	3	3	6
6	5	3	3	2	2	2	3	1	2
6	2	2	4	1	2	1	3	3	6
2	3	4	1	3	2	4	1	4	1
3	3	6	2	4	5	1	4	5	6
4	2	3	2	3	4	2	3	5	4

- MCS solution = 17 / 100 = 0.17

Monte Carlo Simulation (MCS)

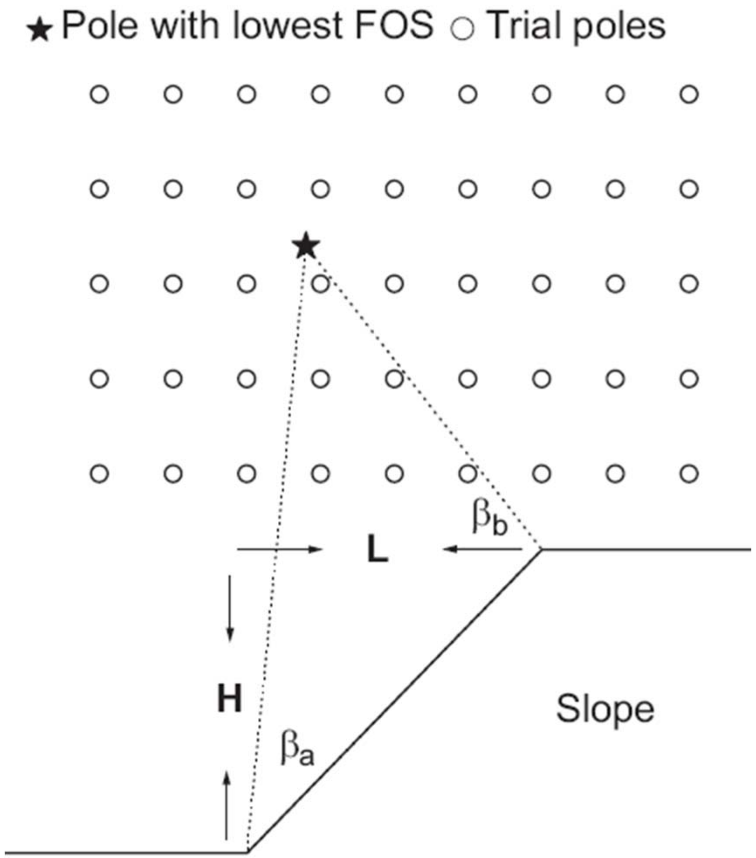
- The larger the sample size, the more reliable the result
- Although there is some suggestion about MCS's sample size from theoretical points of view, sizes such as 50,000 are good enough
- As a result, it is lots of calculation
- To improve the computational efficiency in MCS:

  1) more powerful hardware

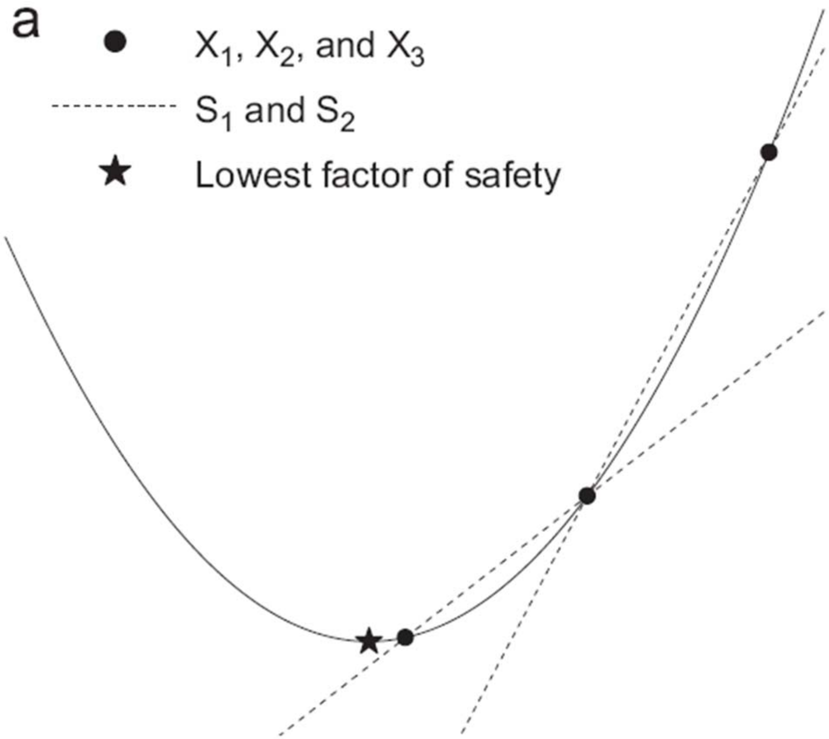
  2) better software (algorithms)

An efficient algorithm in searching for the "pole" in circular slope stability

Trial and Error:



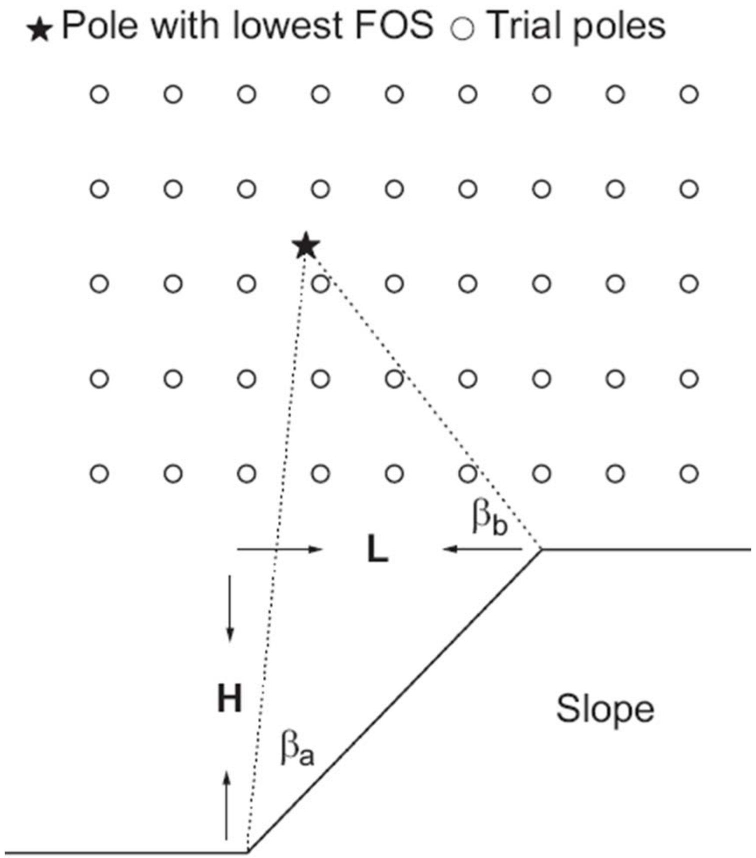
New algorithm



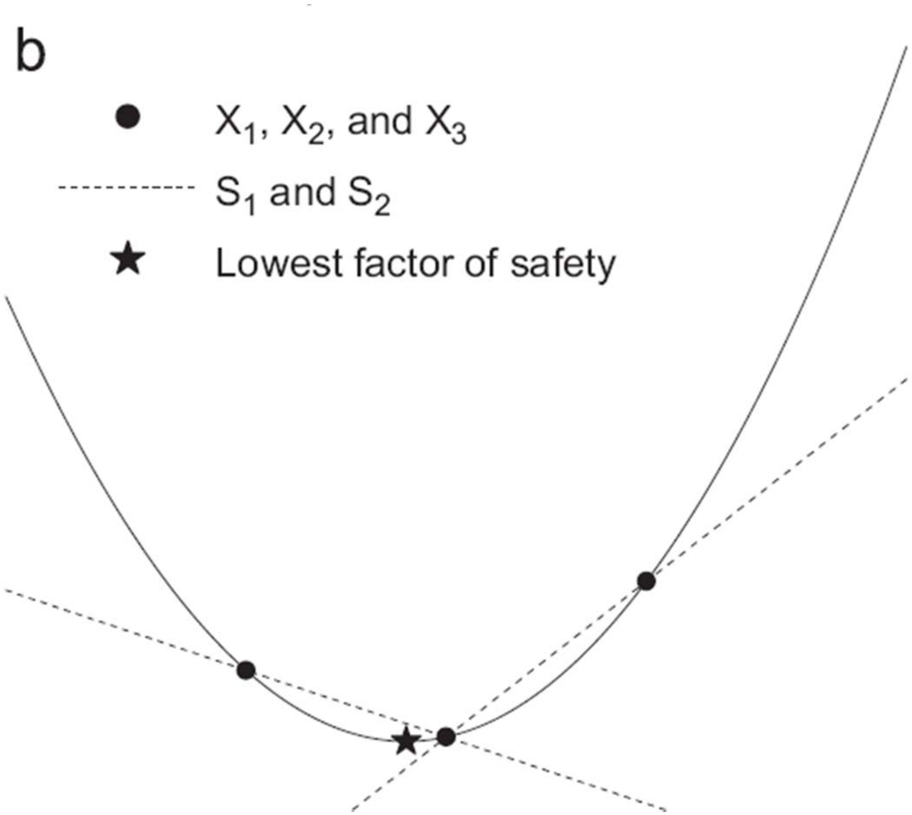
J.P. Wang et al., Computers & Geosciences (2013a)

An efficient algorithm in searching for the "pole" in circular slope stability

Trial and Error:



New algorithm



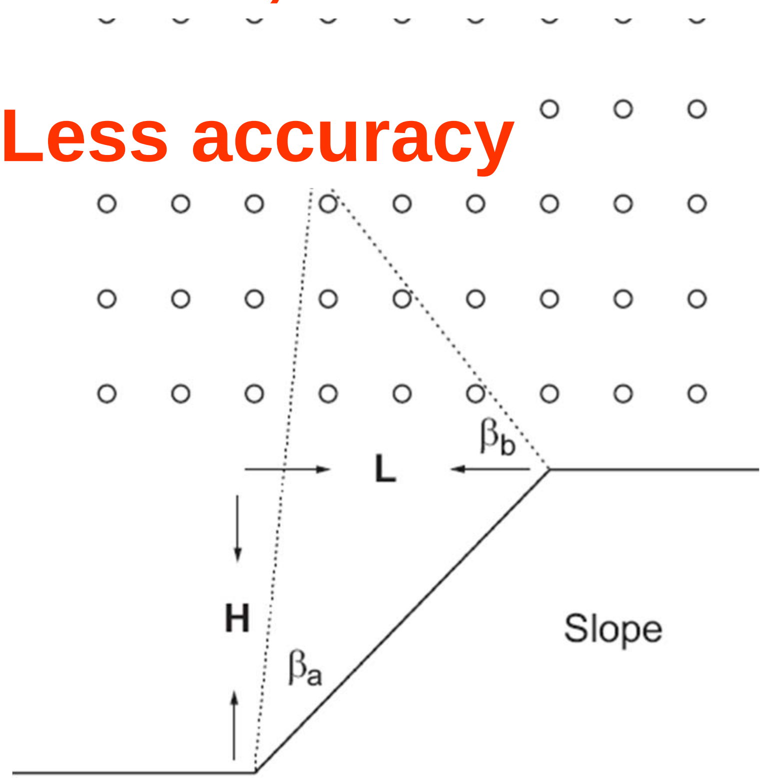
J.P. Wang et al., Computers & Geosciences (2013a)

An efficient algorithm in searching for the "pole" in circular slope stability

Trial and Error:

Ex: in 1,000 iterations

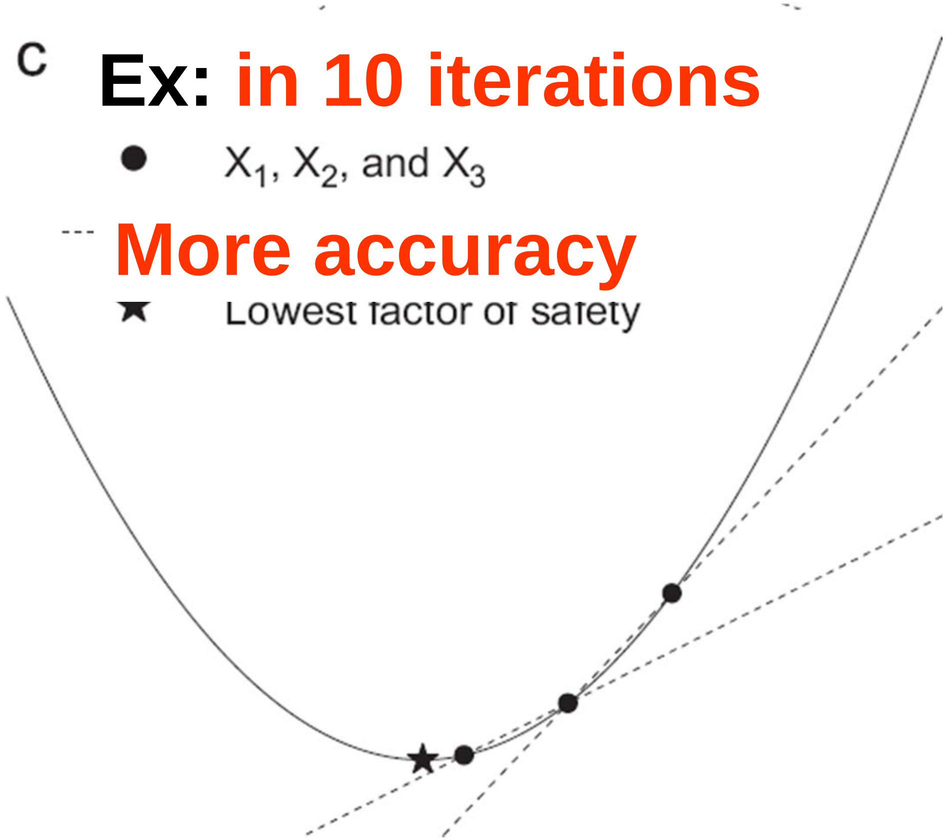
Less accuracy



New algorithm

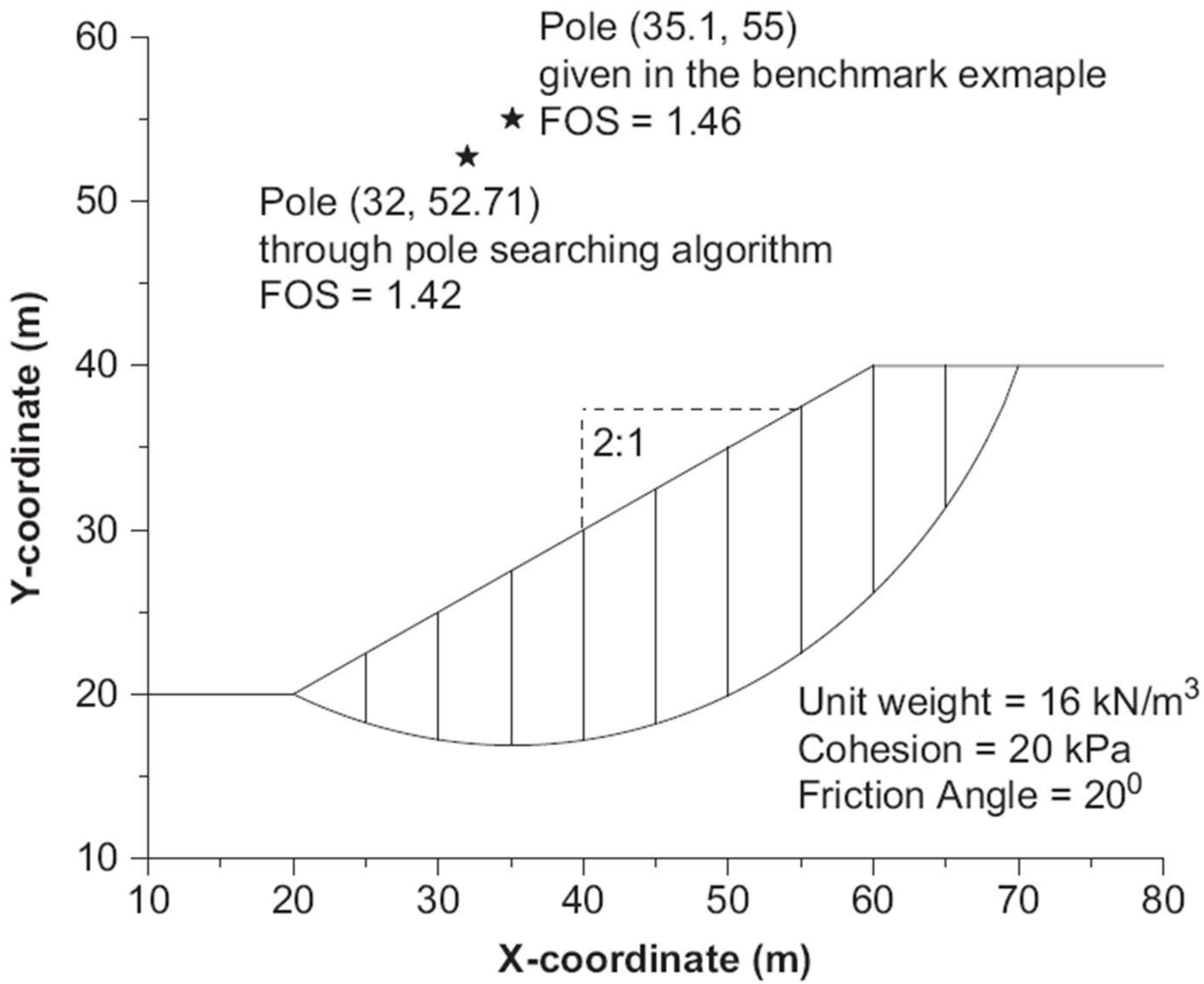
Ex: in 10 iterations

More accuracy



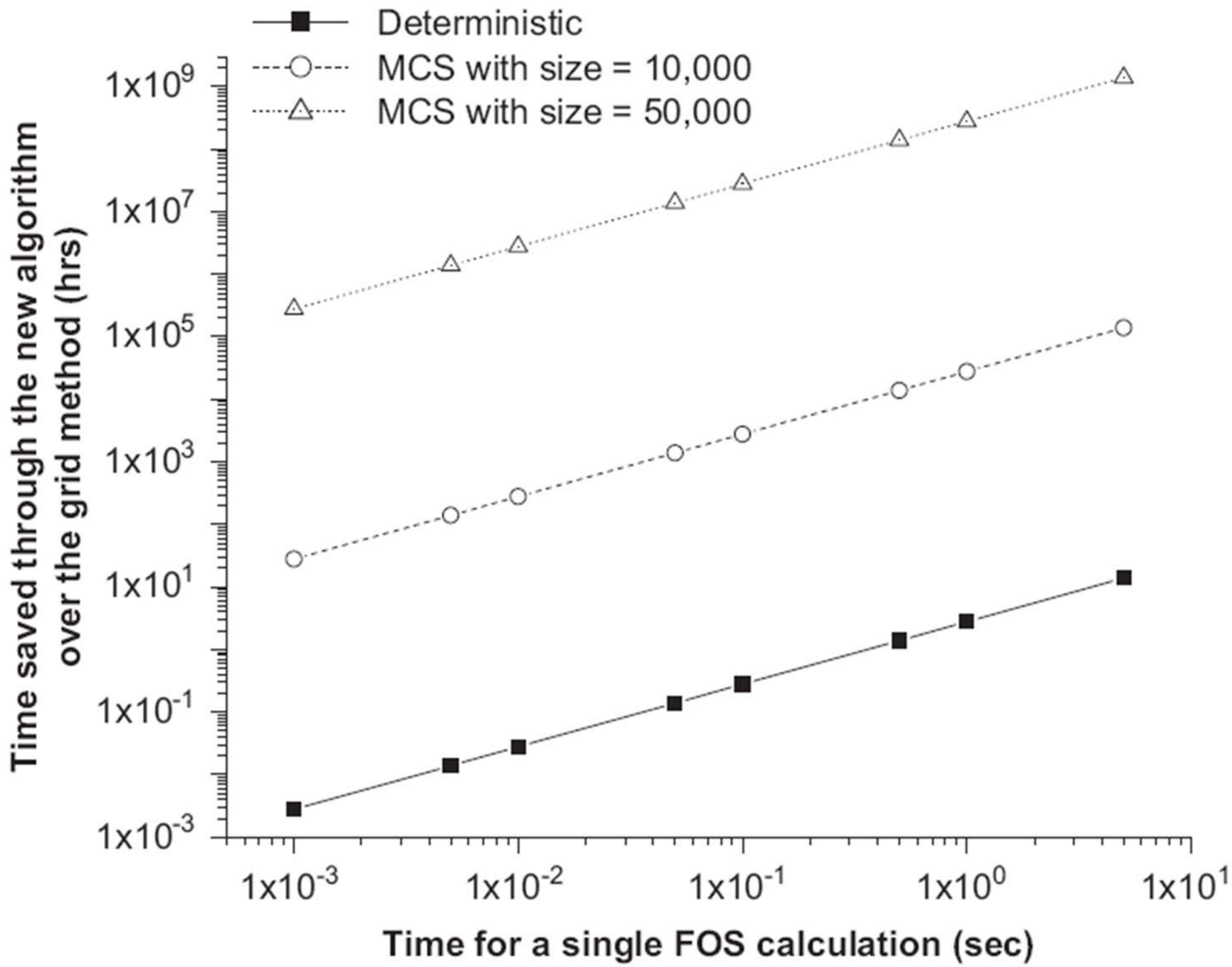
J.P. Wang et al., Computers & Geosciences (2013)

Testing example:



J.P. Wang et al., Computers & Geosciences (2013a)

Testing example



J.P. Wang et al., Computers & Geosciences (2013a)

Rosenblueth method

- When the number of random variables in a problem is not too large (say less than 10), Rosenblueth method is less computational expensive compared to MCS

- Given the number of random variables = **10**, this method still requires $\mathbf{2^{10}}$ iterations (deterministic analysis = 1, MCS = 50,000)

- Considering cohesion and friction angle as random variables, there are four combinations in point estimates

Input		Output
c+	Φ+	FOS1
c+	Φ-	FOS2
c-	Φ+	FOS3
c-	Φ-	FOS4

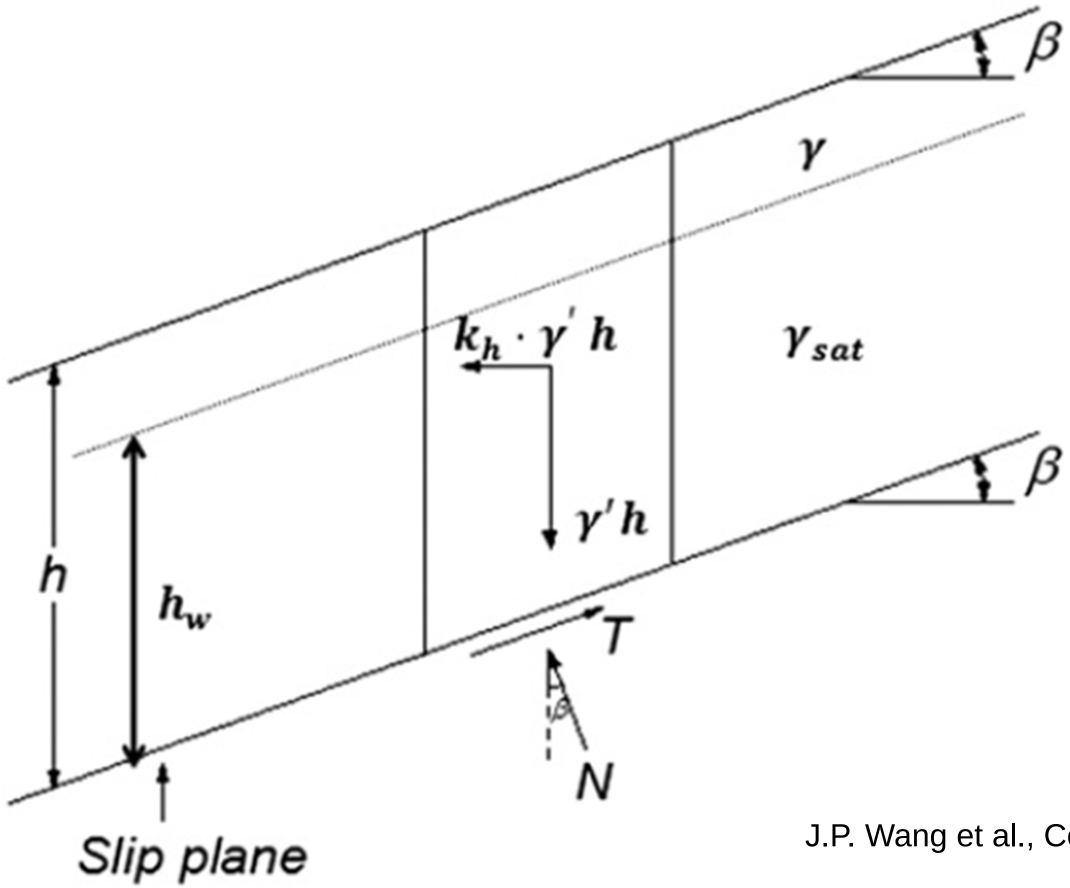
"+" => mean + SD

"–" => mean - SD

Rosenblueth method: testing example

Statistics of the eight parameters of an infinite slope; the values in the parenthesis were used in sensitivity analysis.

	k_h (−)	c (kN/m^2)	ϕ (°)	h (m)	γ_{sat} (kN/m^3)	β (°)	h_w (m)	γ (kN/m^3)
Mean	0.25	20	35	10	26	10	8	20
S.D.	0.1 (0.2)	2 (4)	3.5 (7)	0.5	2.6	0.01	0.8	2
Correlation	$h-h_w$	$\gamma_{sat}-\gamma$	$c-\phi$	other 25 correlations				
	0.8	0.9	0 (−0.7)	0				



J.P. Wang et al., Computers & Geosciences (2012a)

Rosenblueth method: testing example

- Automatically generate the following matrix:

	X1	X2	X3	X4	X5	X6	X7	X8
Trial 1	+	+	+	+	+	+	+	+
Trial 2	+					+	+	-
Trial 3	+					+	-	+
Trial 4	+					+	-	-
Trial 5	+					-	+	+
Trial 6	+					-	+	-
Trial 7	+					-	-	+
Trial 8	+					-	-	-
...								
...								
...								
Trial 256	-					-	-	-

$$m = i - 1$$

$$k = 2^{4-i}$$

Rosenblueth method: testing example

- Automatically generate the following matrix:

RosenPoint: Print Title

RosenPoint: Matrix Generation

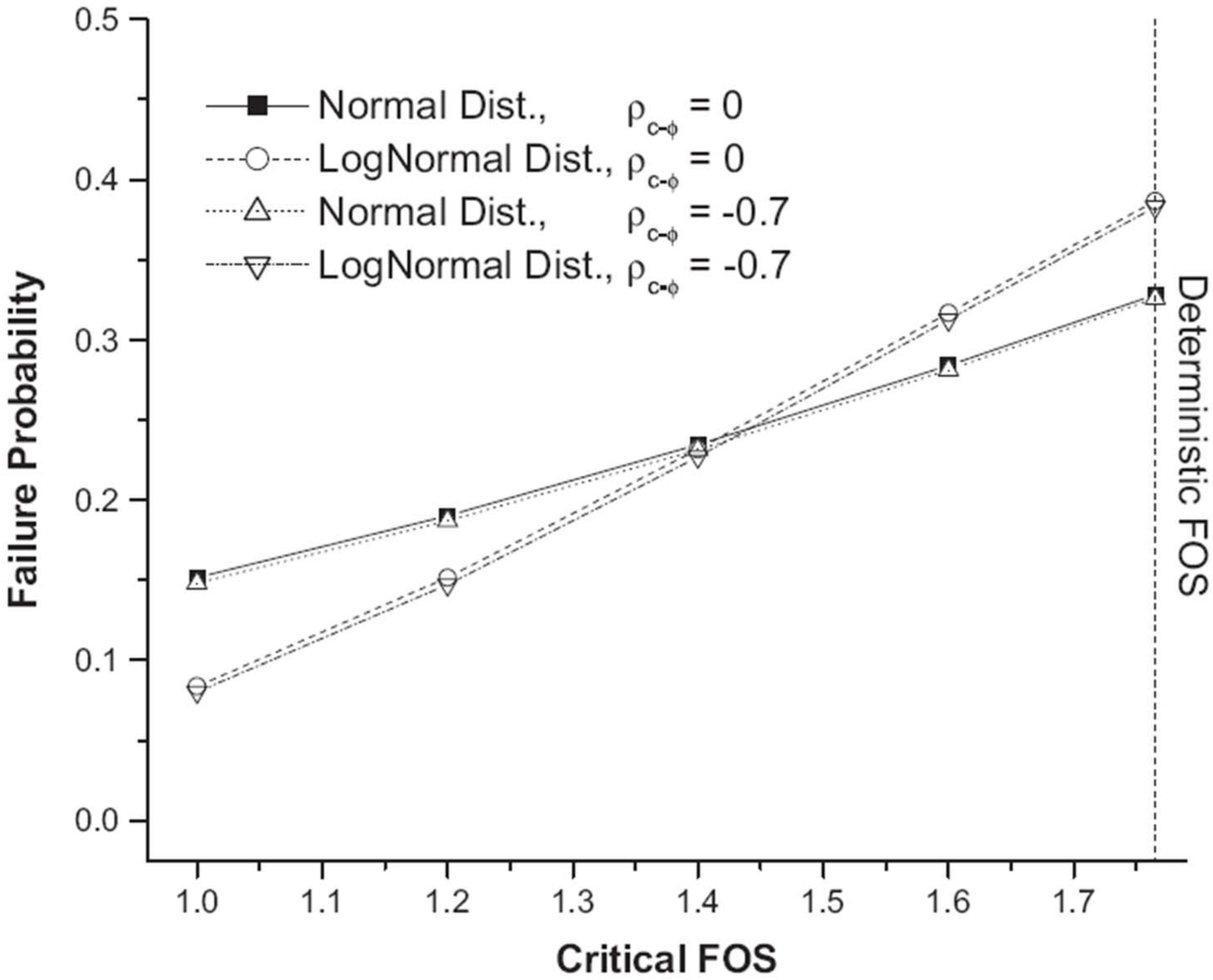
RosenPoint: FOS and Pf Computation

	a	b	c	d		
MEAN	0.25	20	35	20		
S.D.	0.2	5	15	2		
	a-b	a-c	a-d	b-c	b-d	c-d
Correlation	0	0	1	1	0	0.5

a	b	c	d	Weight	FOS		Mean_F	SD_F	Cri_F	PF~N	PF~LogN
0.45	25	50	22	0.21875							
0.45	25	50	18	0.03125							
0.45	25	20	22	0.03125							
0.45	25	20	18	-0.03125							
0.45	15	50	22	0.09375							
0.45	15	50	18	-0.09375							
0.45	15	20	22	0.15625							
0.45	15	20	18	0.09375							
0.05	25	50	22	0.09375							
0.05	25	50	18	0.15625							
0.05	25	20	22	-0.09375							
0.05	25	20	18	0.09375							
0.05	15	50	22	-0.03125							
0.05	15	50	18	0.03125							
0.05	15	20	22	0.03125							
0.05	15	20	18	0.21875							

J.P. Wang et al., Computers & Geosciences (2012a)

Rosenblueth method: testing example



J.P. Wang et al., Computers & Geosciences (2012a)

Seismic hazard analysis

- To best estimate a design ground motion (e.g., PGA = 0.3 g), or the rate of motion of exceedance (e.g., PGA > 0.3 g = 0.01 per year)

- Two representative methods:

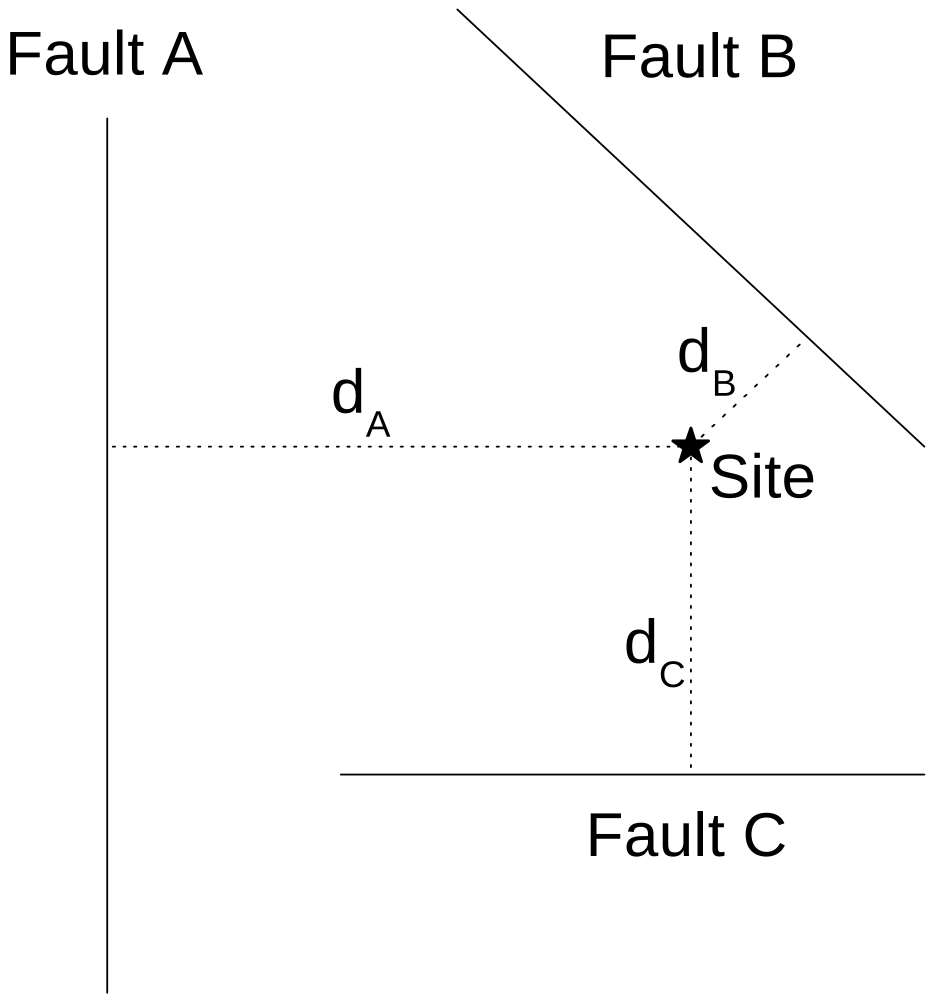
  1) Deterministic Seismic Hazard Analysis (DSHA)

  => worse-case earthquake size and location, mean value from ground motion models

  2) Probabilistic Seismic Hazard Analysis (PSHA)

  => accounting for the uncertainties of size, location, ground motion models

Overview of DSHA

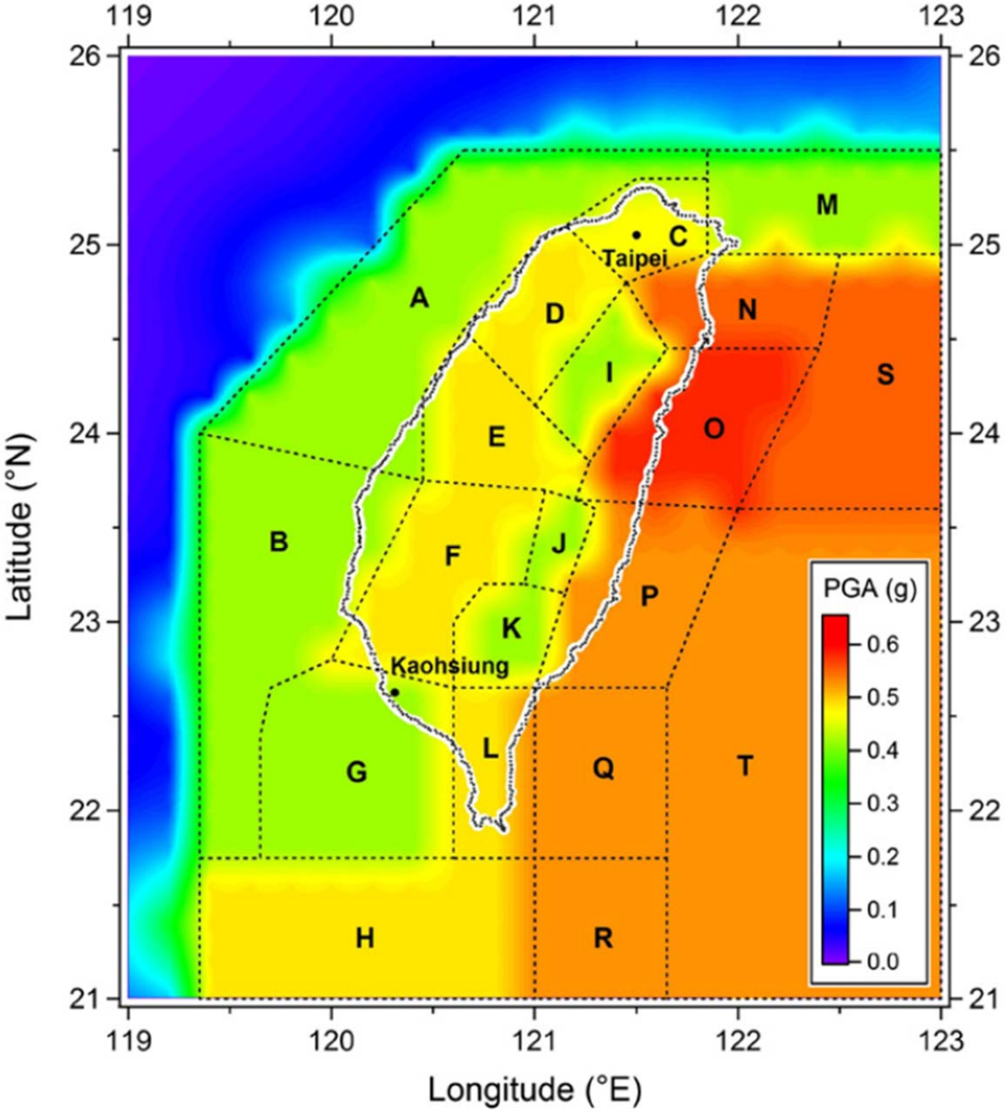


$$DSHA = MAX\ \{H_A, H_B, H_C\}$$

$$H_A = f(m_A, d_A)$$
$$H_B = f(m_B, d_B)$$
$$H_C = f(m_C, d_C)$$

m_A, m_B and m_C are the best estimate of maximum earthquakes induced by the fault

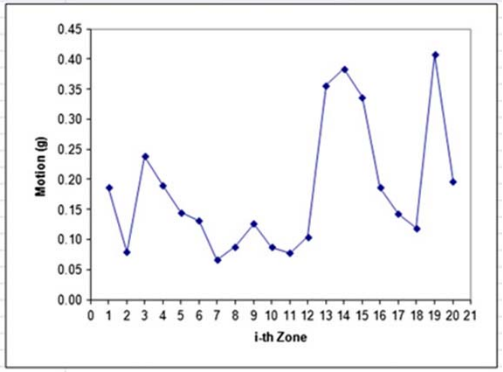
DSHA map for Taiwan



	AA	AB	AC	AD	AE	AF	AG	AH
5	Site		Magnitude	Motion				
6	Long.	Lat.	Increment	above SD				
7	123	26	0	0	1. Heading			
8								
9	No. Source	20						
10	Zone 1		Zone 2		Zone 3		Zone 4	
11	MCS	6.5	MCS	6.5	MCS	7.1	MCS	7.3
12	Zone Boundary		Zone Boundary		Zone Boundary		Zone Boundary	
13	Longitude	Latitude	Longitude	Latitude	Longitude	Latitude	Longitude	Latitude
14	120.65	25.5	119.35	24	121.85	25.35	121.15	25.1
15	121.85	25.5	120.45	23.75	121.85	24.95	121.45	24.8
16	121.85	25.35	120	22.8	121.45	24.8	121	24.15
17	121.5	25.35	119.7	22.65	121.15	25.1	120.65	24.55
18	121.15	25.1	119.65	22.4	121.5	25.35	121	25
19	121	25	119.65	21.75				
20	120.65	24.55	119.35	21.75				
21	120.45	24.2						
22	120.45	23.75						
23	119.35	24						

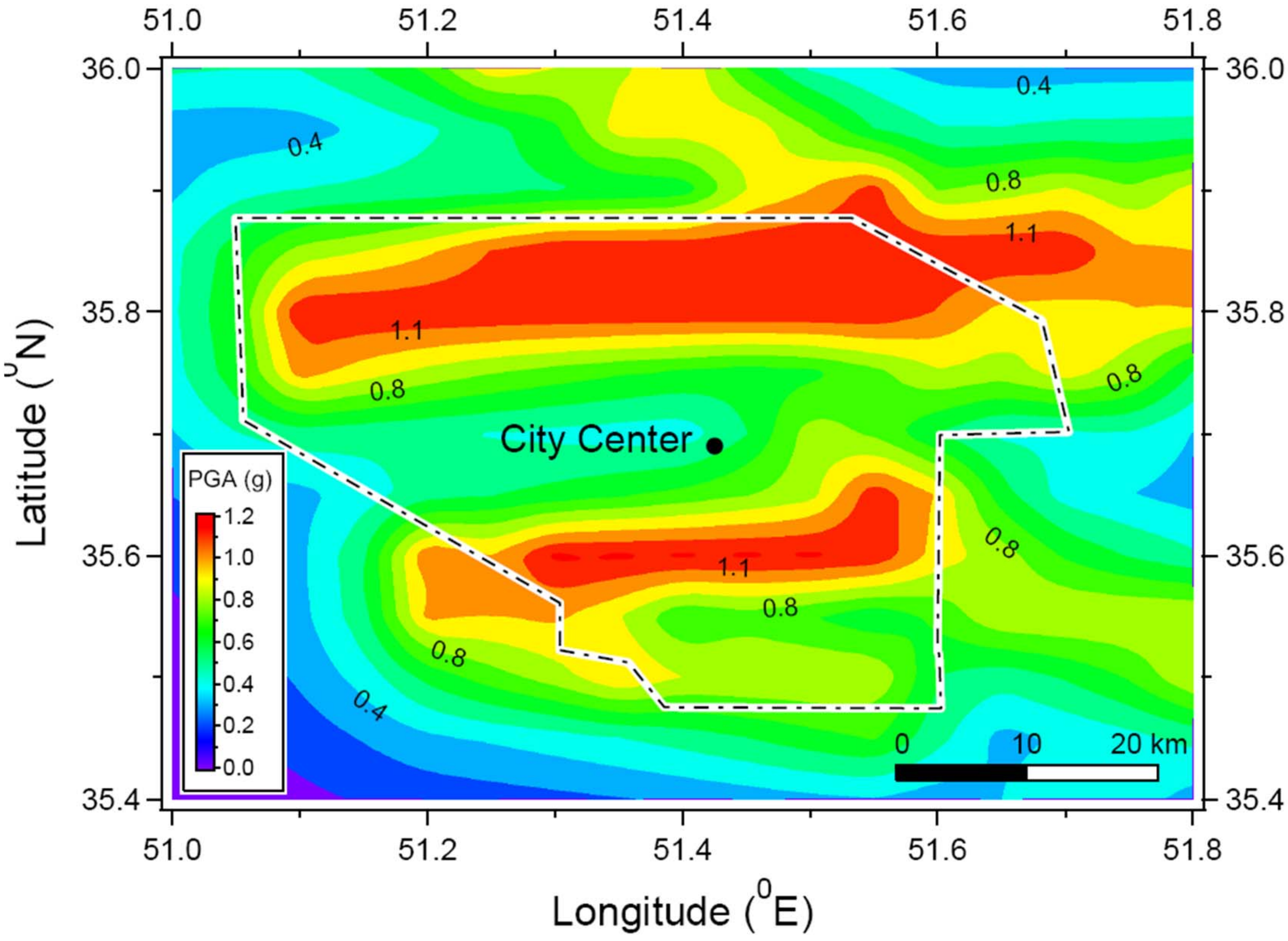
	U	V	W	X
6				
7				
8			DSHA	0.41
9				
10	Zone	MCS	Dist (km)	Motion (g)
11	1	6.50	127.89	0.19
12	2	6.50	358.82	0.08
13	3	7.10	136.04	0.24
14	4	7.30	205.04	0.19
15	5	7.30	285.34	0.14
16	6	7.30	322.66	0.13
17	7	6.50	444.82	0.07
18	8	7.30	514.45	0.09
19	9	6.50	205.04	0.13
20	10	6.50	317.26	0.09
21	11	6.50	367.99	0.08
22	12	7.30	424.03	0.10
23	13	6.50	55.60	0.36
24	14	8.00	127.09	0.38
25	15	8.30	182.61	0.34
26	16	7.80	285.32	0.19
27	17	7.80	396.81	0.14
28	18	7.80	492.10	0.12
29	19	8.00	116.75	0.41
30	20	7.80	266.81	0.20

2 . DSHA (single site)



J.P. Wang et al., Computers & Geosciences (2012b)

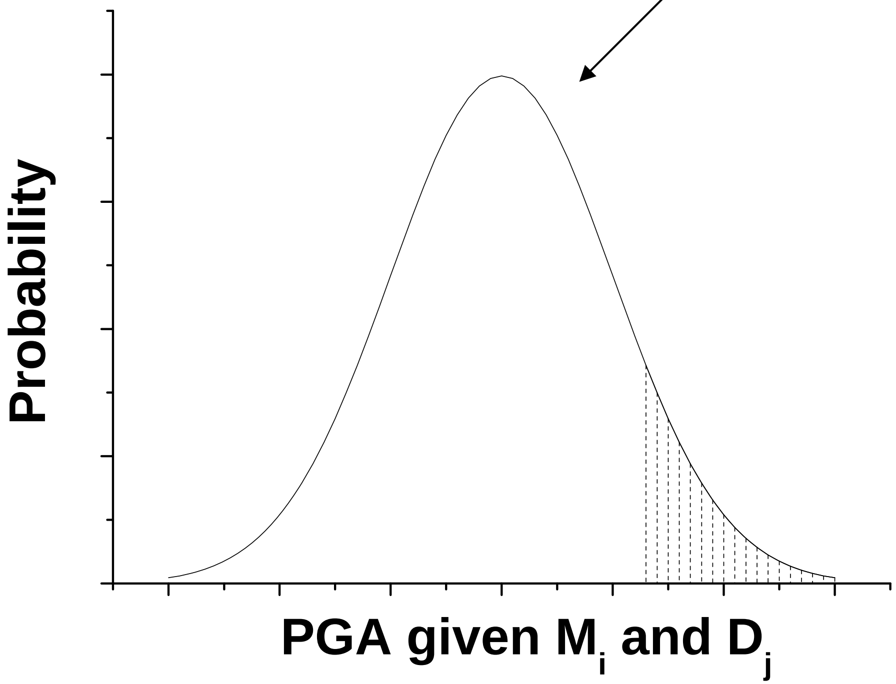
DSHA map for Tehran

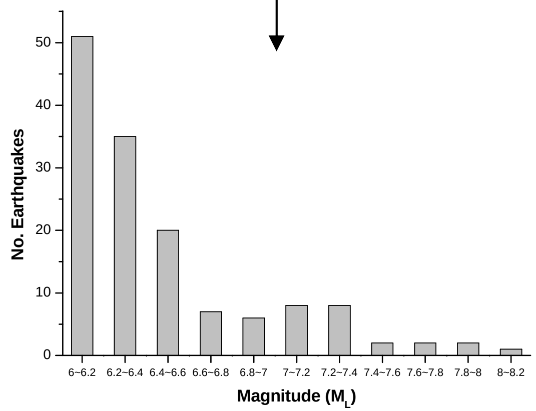


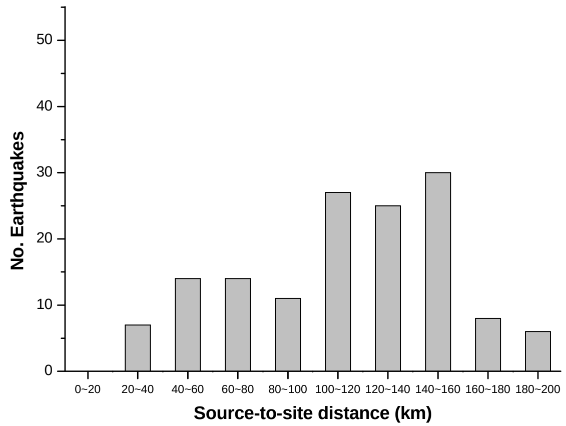
J.P. Wang and H. Taheri, Natural Hazards Review ASCE (in press)

Overview of PSHA

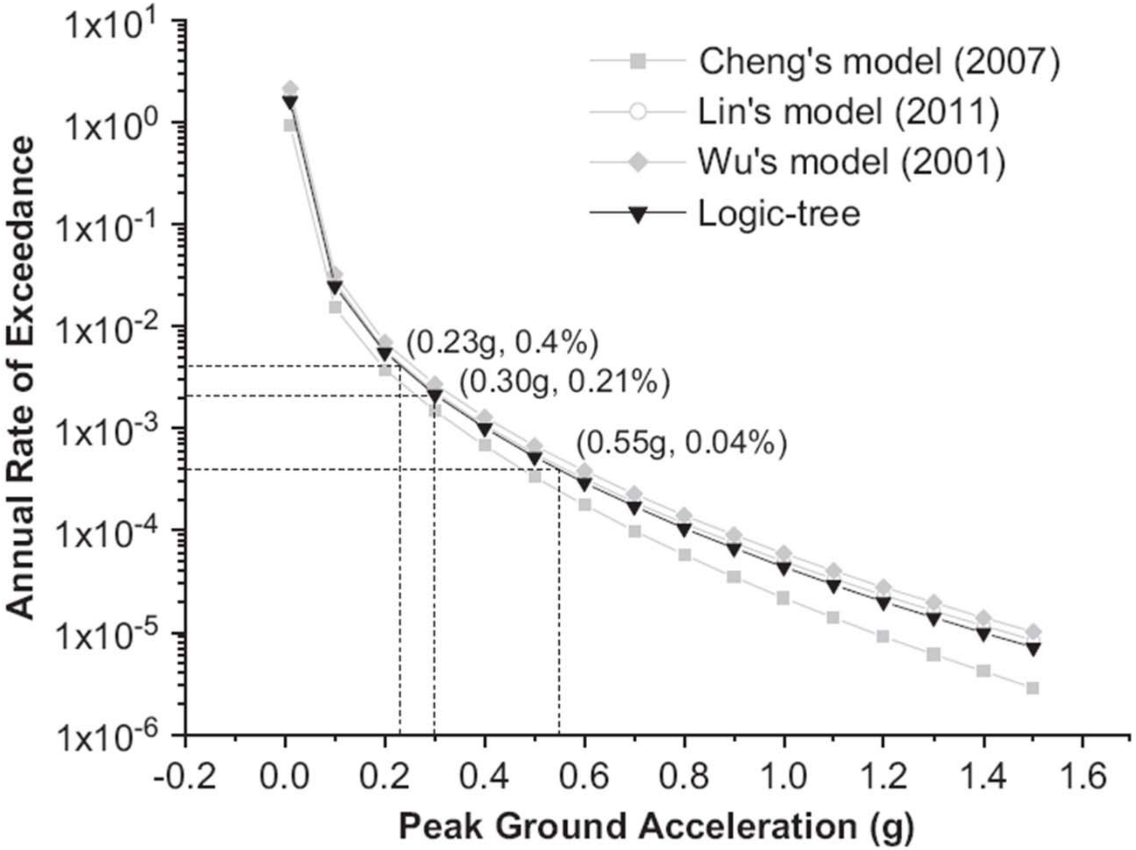
$$\lambda(Y > y^*) = \sum_{i=1}^{N_S} \nu_i \sum_{j=1}^{N_M} \sum_{k=1}^{N_D} \Pr[Y > y^* \,|\, m_j, d_k] \times \Pr[M = m_j] \times \Pr[D = d_k]$$

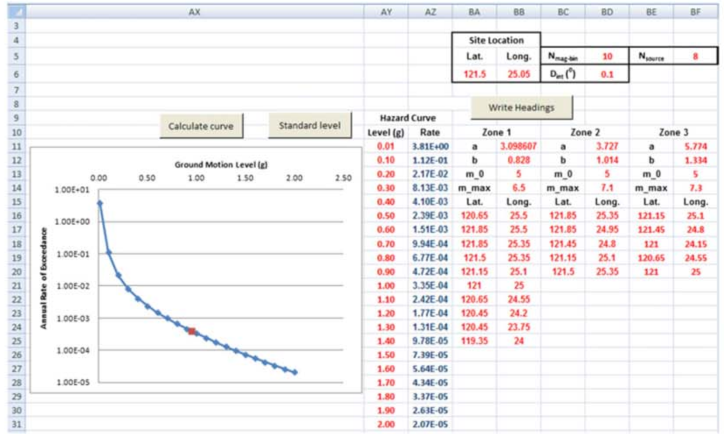


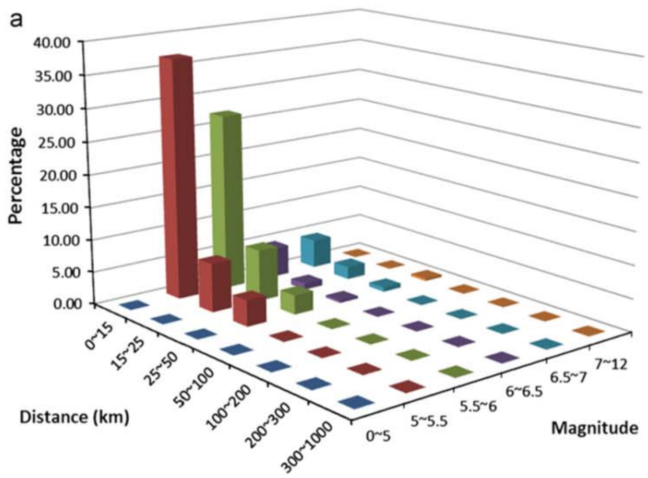




PSHA study for Taipei



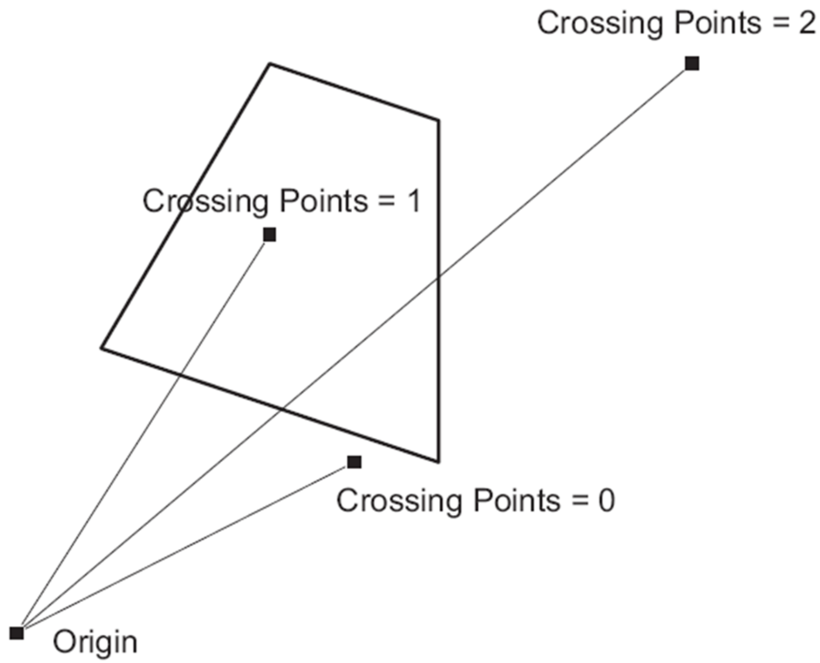




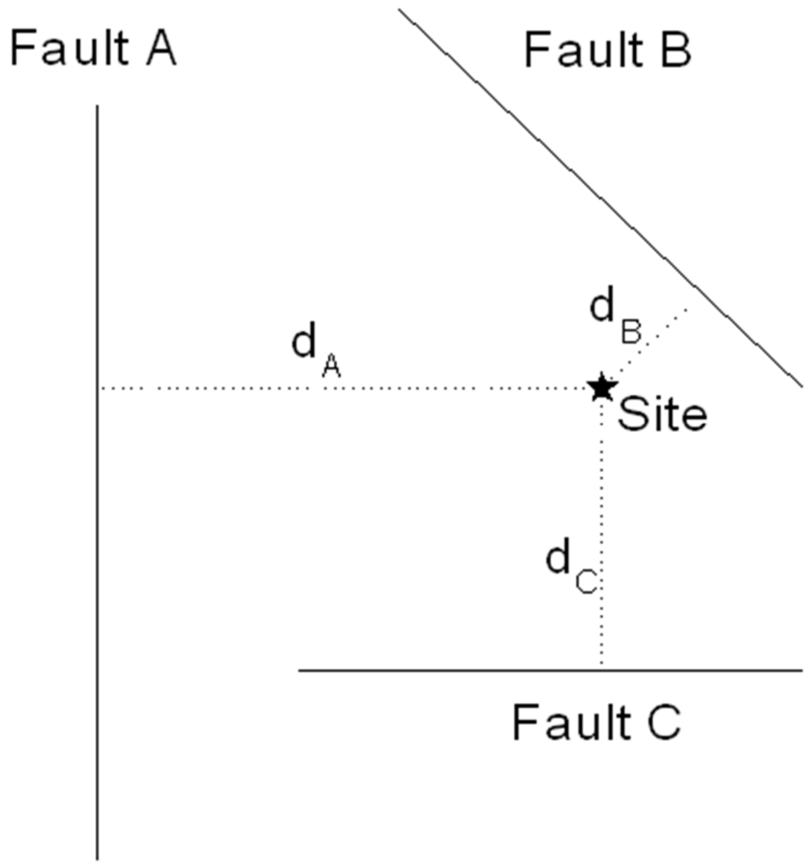
J.P. Wang et al., Computers & Geosciences (2013b)

Notes on DSHA and PSHA tools

- The algorithms seem not that challenging
- One interesting and challenging algorithm, needed but not reflected on the governing equations, is to determine a point inside a given polygon or not



A modified DSHA



Scenario A:
[0.5, 0.1, 0.1]

Scenario B:
[0.5, 0.5, 0.5]

DSHA not accounting for the hazards from non-controlling sources

A modified DSHA

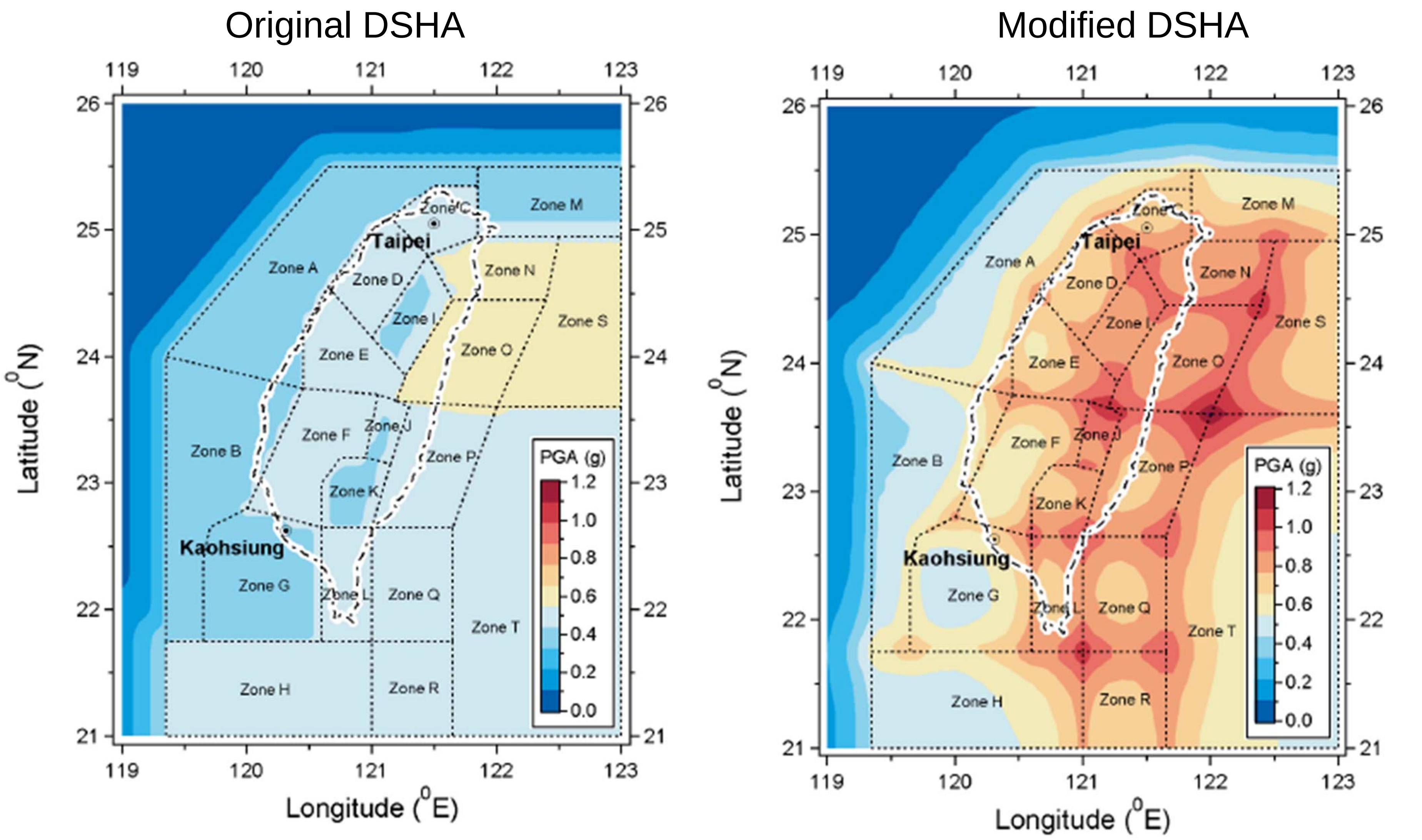
- Extreme probability theory: the distribution of extreme values: i.e., maximum and minimum

$$\tilde{Y} = \max\{Y_1, \cdots, Y_n\}$$

to compute $\Pr(\tilde{Y} \leq y_{50}) = 0.5$

- For comparing it to DSHA, we calculate the estimate at the same percentile

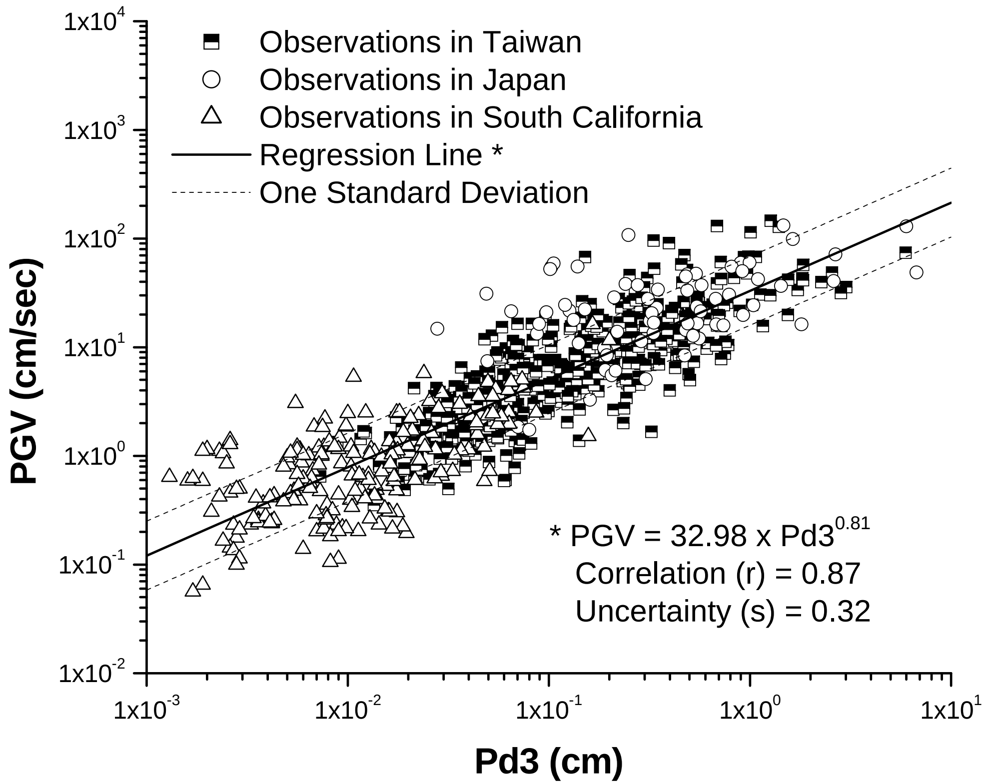
J.P. Wang and D. Huang, Bulletin of Engineering Geology and the Environment (in press)

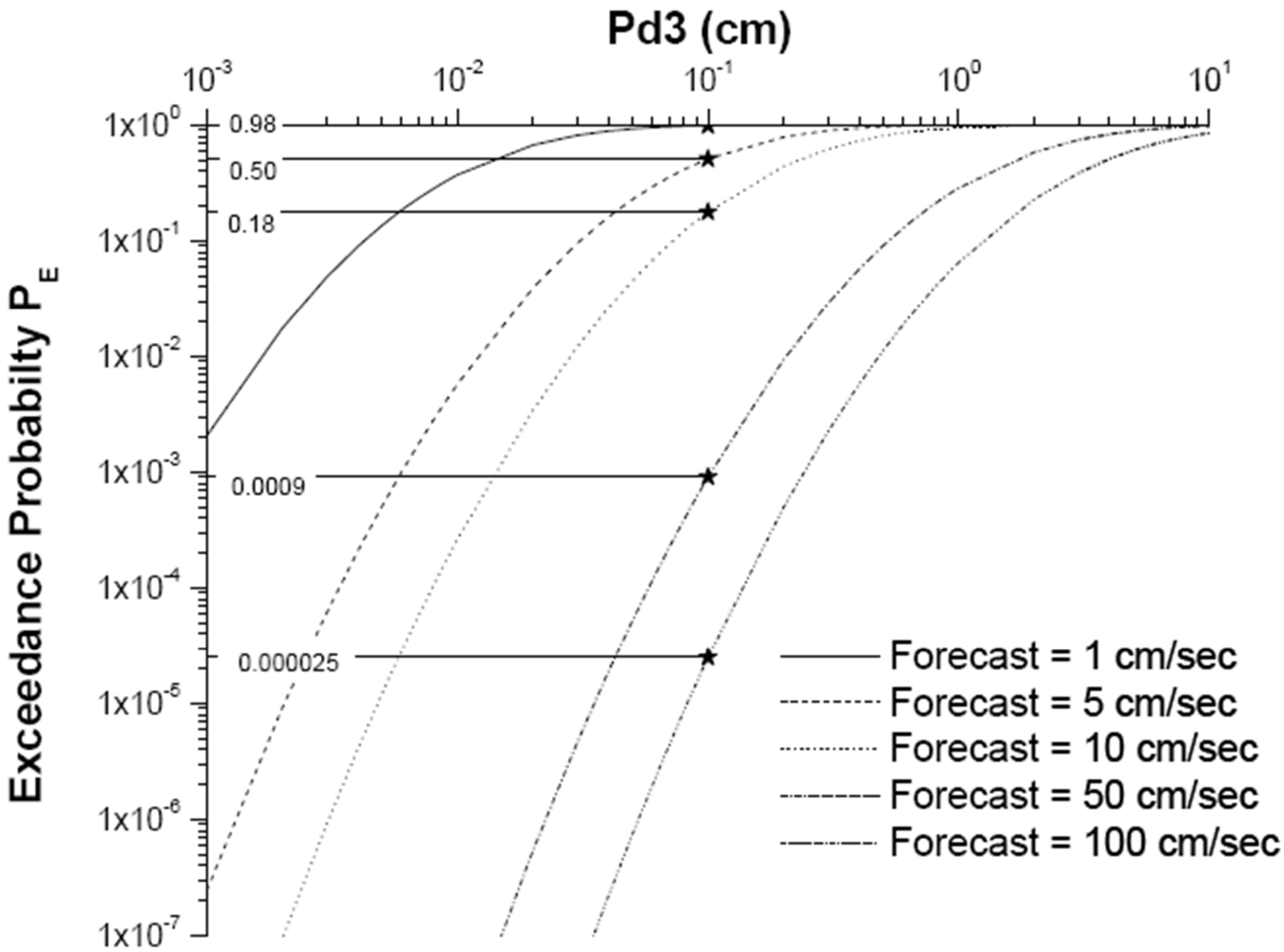


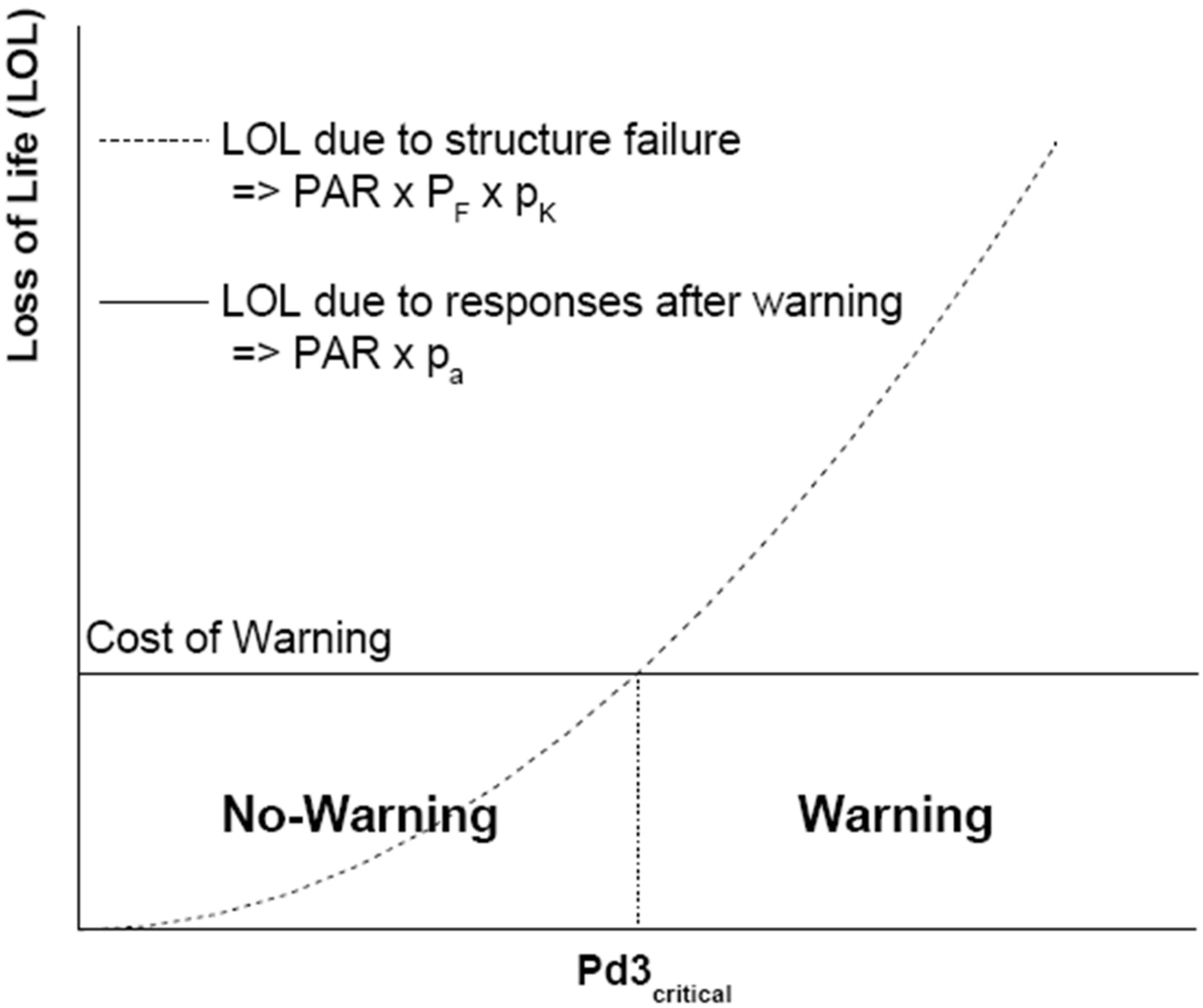
J.P. Wang and D. Huang, Bulletin of Engineering Geology and the Environment (in press)

Regression analysis and Excel

- Regression analysis is widely used for finding some empirical relationships




Pd3 (cm)
10-3
10-2
10-1
10^0
10^1
Exceedance Probabilty P_E
1x10^0
1x10-1
1x10-2
1x10-3
1x10-4
1x10-5
1x10-6
1x10-7
0.98
0.50
0.18
0.0009
0.000025
Forecast = 1 cm/sec
Forecast = 5 cm/sec
Forecast = 10 cm/sec
Forecast = 50 cm/sec
Forecast = 100 cm/sec



J.P. Wang et al. (2012c) Soil Dynamics and Earthquake Engineering

Matrix calculations and Excel

- Matrix-based analysis such as multiple regression analysis is adopted for geosciences problems
- However, using those Excel functions on spreadsheets is not very convenient
- SHIFT + CONTROL + ENTER

- Using those functions in macros can save some hassle

- Ex: matrix multiplication

  c = MMULT(a, b)

- A multiple-regression toolkit was developed with such a simple alternative, which was then applied to earthquake early warning studies

J.P. Wang et al. (2013c) Computers and Geosciences

Eigan values and Eigan factors

- Calculating eigan values of a matrix is the key to some statistical analyses, such as the principal-component analysis

- But I so far have no better solutions to calculate such a problem in Excel

Conclusions and Discussions

- Probabilistic analysis is a calculation that considers the variance of inputs, while deterministic analysis only considers mean values

- Excel should be a good option to probabilistic analysis