# The Application of the Statistical Mechanical Model of Earthquake

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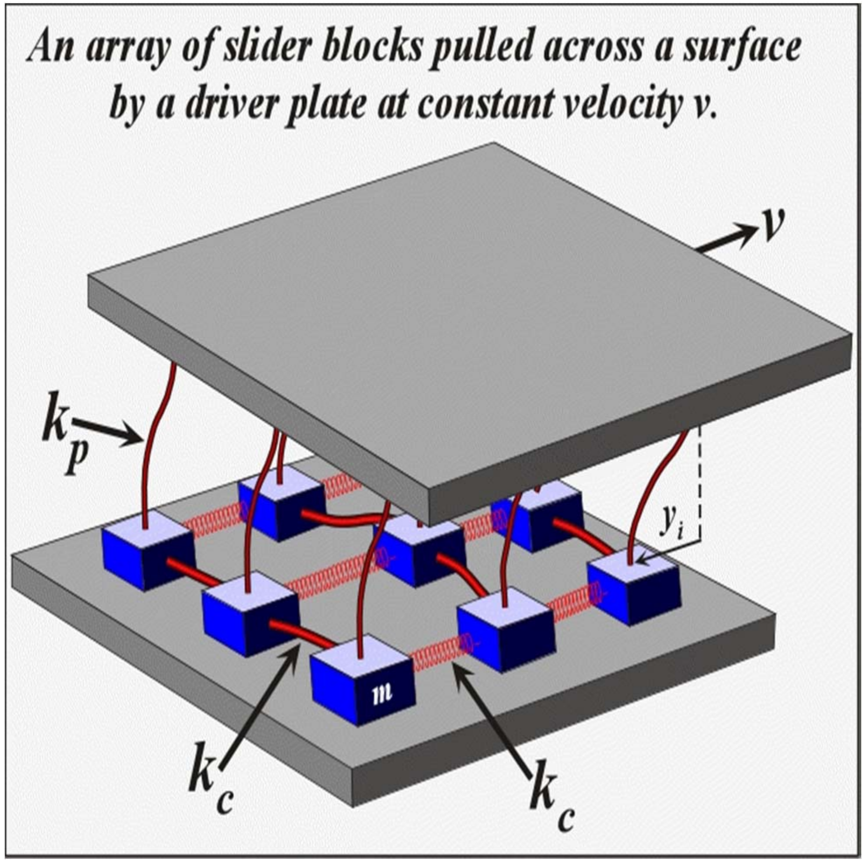
# Outline

- Introduction
- Long-Range Connective Sandpile Model
  - Intermittent Criticality
  - B-Value decay
  - B-Value and Hurst Exponent
  - Quasi-Periodicity
- Conclusion

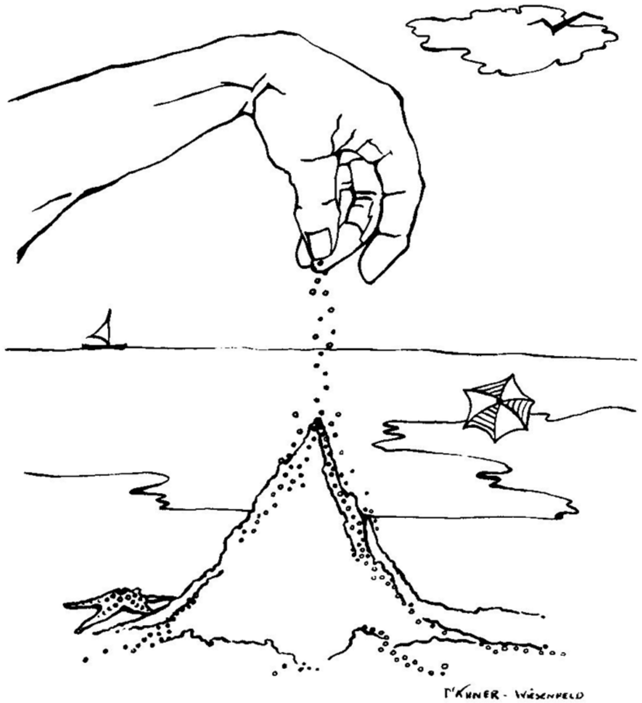
# Statistical Numerical Models

## Slider-Blocks Model




[Burridge and Knopoff, 1967]

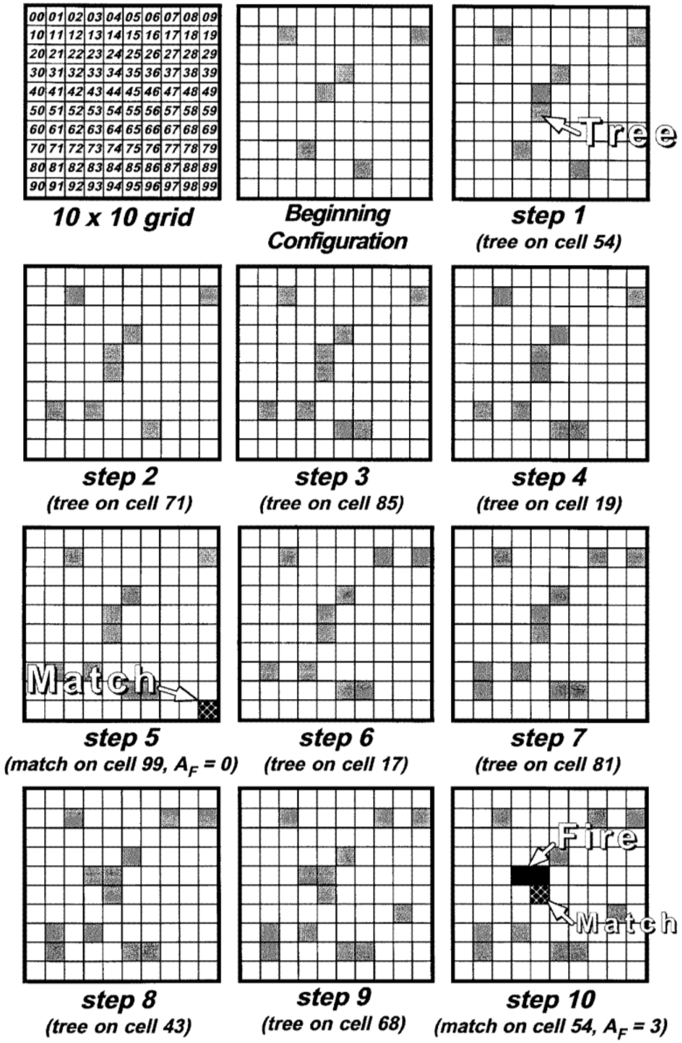
## Sandpile Model



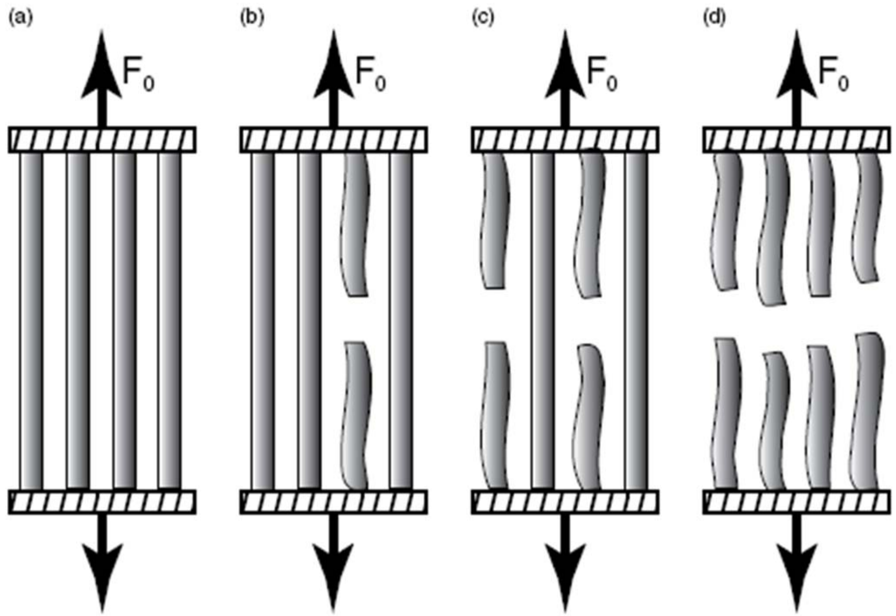
[Bak, Tang, and Wiesenfeld, 1987; 1989]

# Statistical Numerical Models

## Forest-Fire Model




## Fiber-Bundle Model (Damage mechanical model)




[Turcotte,GJI, Rep. Prog. Phys., 2003]

# Self-Organized Criticality (SOC)

## Power law

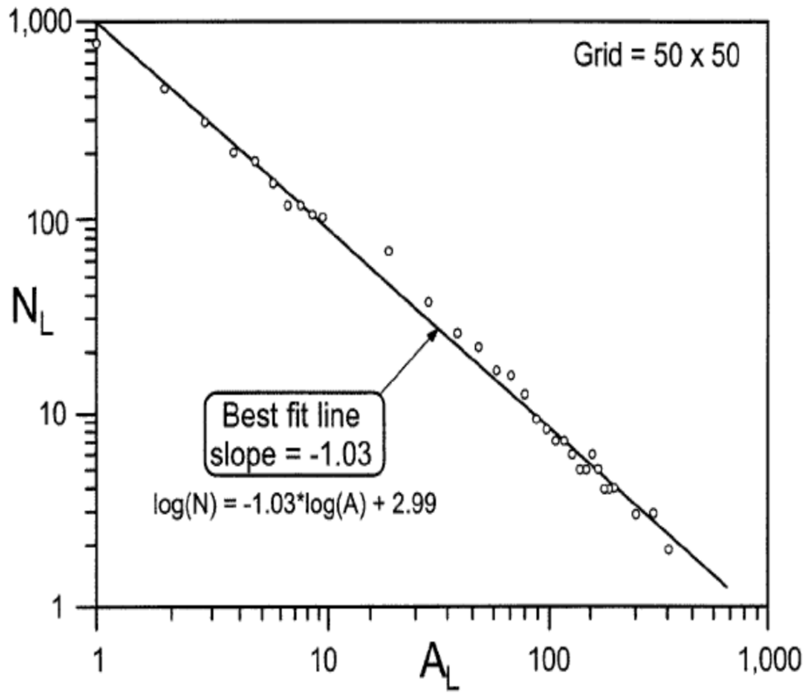
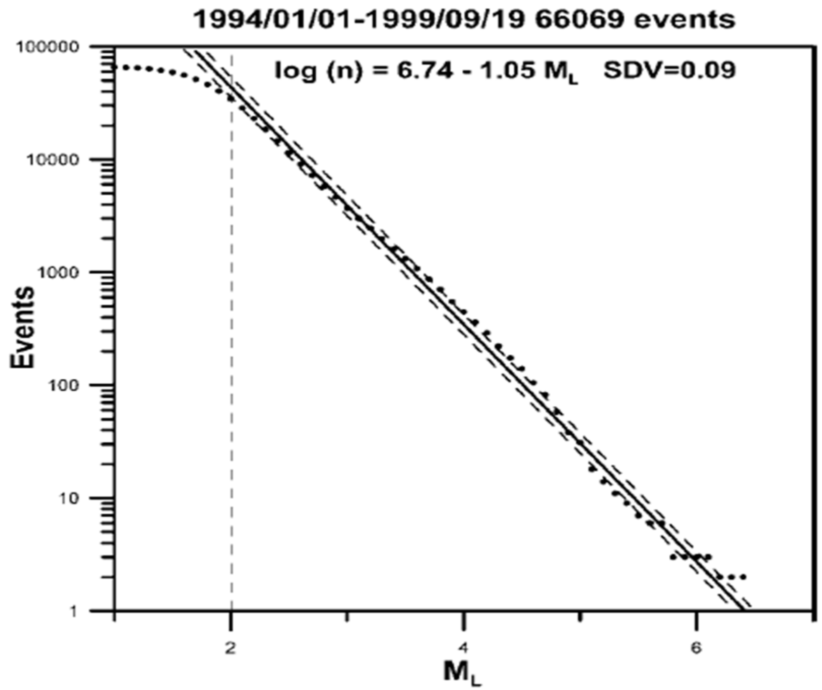



**Figure 2.** Noncumulative frequency–area distribution for a sandpile model on a $50 \times 50$ grid. The number of model events, $N_L$, in which a specified number of boxes, $A_L$, become unstable, is given as a function of $A_L$.

## Gutenberg-Richter law




[Turcotte,GJI, Rep. Prog. Phys., 2003]

# What is SOC?

- The systems organize themselves at the critical point without any significant “tuning” of the system from outside
- The “System” evolves in time under the two influence of:

1. *External driving force or forces;*
2. *Forces of internal interactions.*

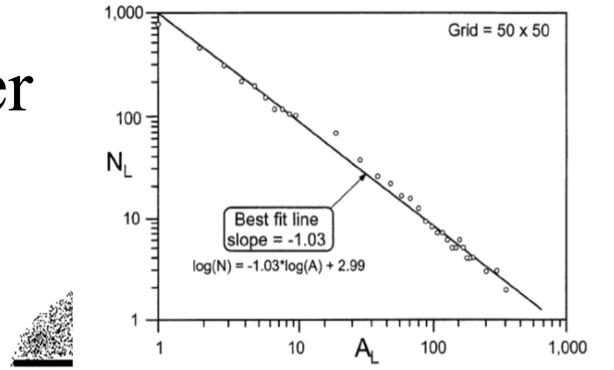



**Figure 2.** Noncumulative frequency–area distribution for a sandpile model on a $50 \times 50$ grid. The number of model events, $N_L$, in which a specified number of boxes, $A_L$, become unstable, is given as a function of $A_L$.

- Power law behavior between event size and which frequency.
- The Idea: we can specify a simplified mechanism that produces a typical behavior of many complex systems.

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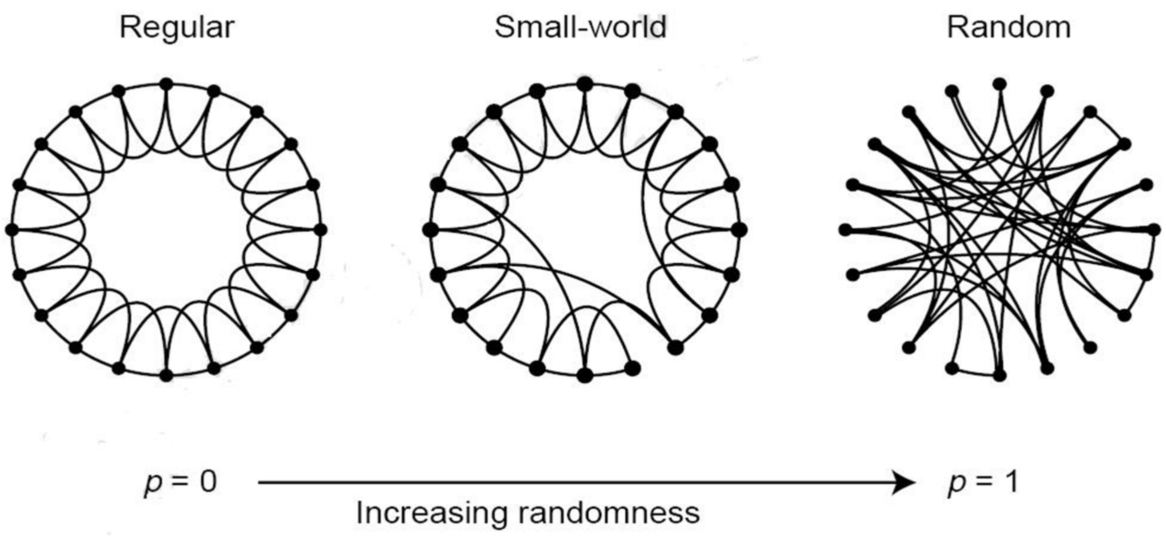
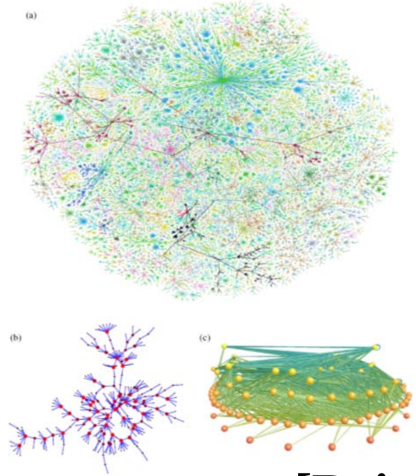
# Small-world Networks

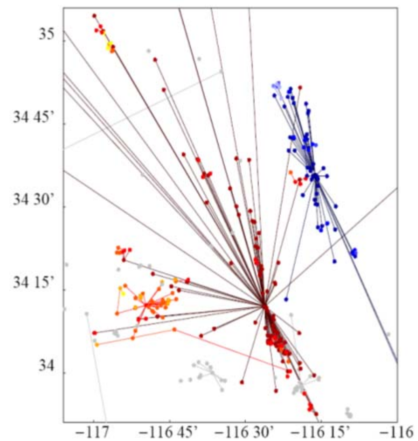



**[D.J. Watts & S.H. Strogatz, *Nature*, 393, 440-442, 1998]**

- Social Networks
- Information Networks
- Biological Networks
- Earthquake Networks

**[Albert and Barabási, 2002]**

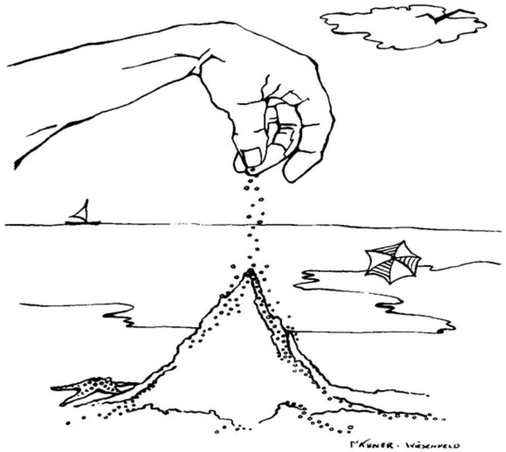


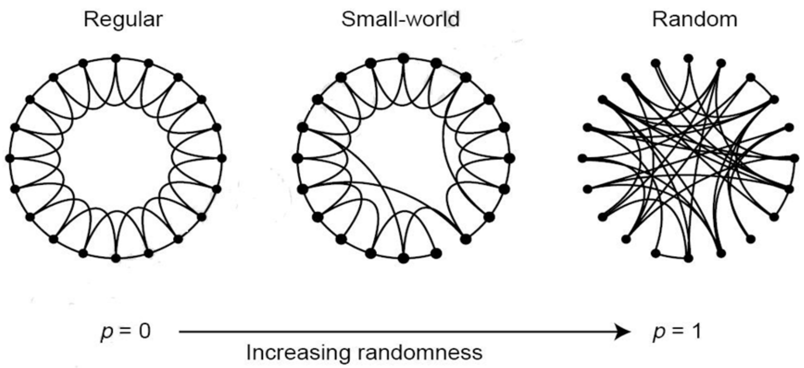


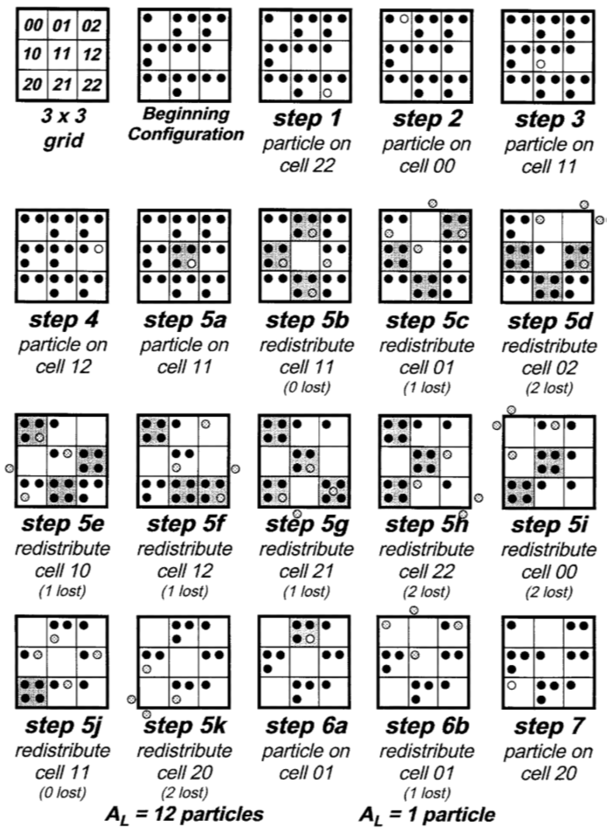

**[Baiesi and Paczuski, 2004]**

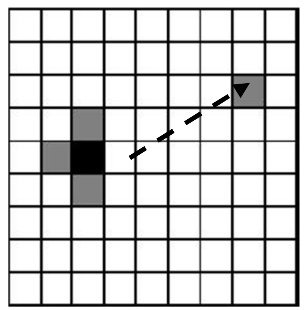
# Collective Dynamics of ‘Small-world’ Networks

## BTW sandpile model





## Long-Range Connective Sandpile Model

# Long-Range Random Connection

- Probability of long-range random connection
  - change of system permeability
  - fingerling channels of pore pressure
  - dynamic triggering of seismic waves

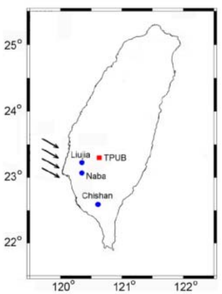
GEOPHYSICAL RESEARCH LETTERS, VOL. 36, L09404, doi:10.1029/2009GL037330, 2009

Click Here for Full Article

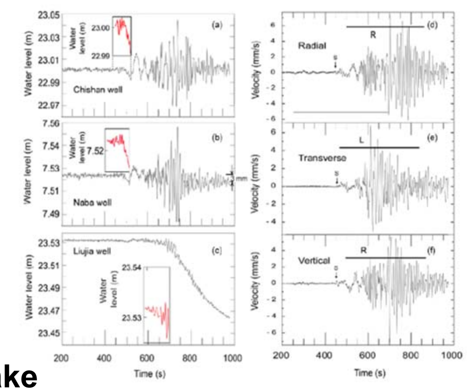
**Role of S waves and Love waves in coseismic permeability enhancement**

Chi-yuen Wang,[1] Yeeping Chia,[2] Pei-ling Wang,[2] and Douglas Dreger[1]

Received 16 January 2009; revised 8 April 2009; accepted 16 April 2009; published 15 May 2009.




2008 M7.9 Wenchuan earthquake

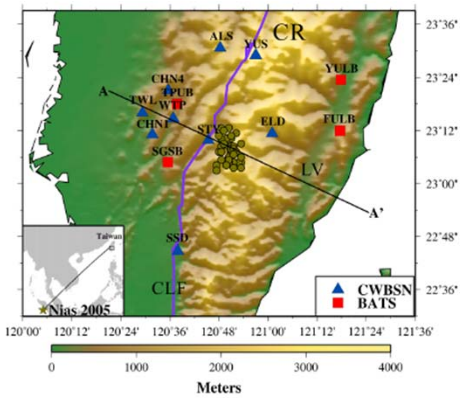


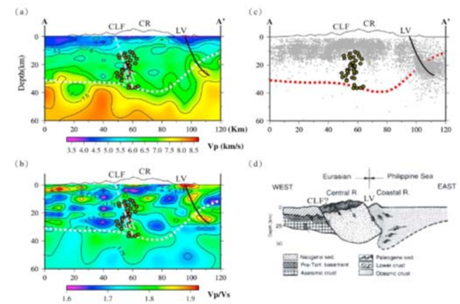

GEOPHYSICAL RESEARCH LETTERS, VOL. 37, L16307, doi:10.1029/2010GL043918, 2010

**Detecting low-frequency earthquakes within non-volcanic tremor in southern Taiwan triggered by the 2005 Mw8.6 Nias earthquake**

Chi-Chia Tang,[1,2] Zhigang Peng,[2] Kevin Chao,[2] Chau-Huei Chen,[1] and Cheng-Horng Lin[3]

Received 17 May 2010; revised 24 June 2010; accepted 30 June 2010; published 25 August 2010.



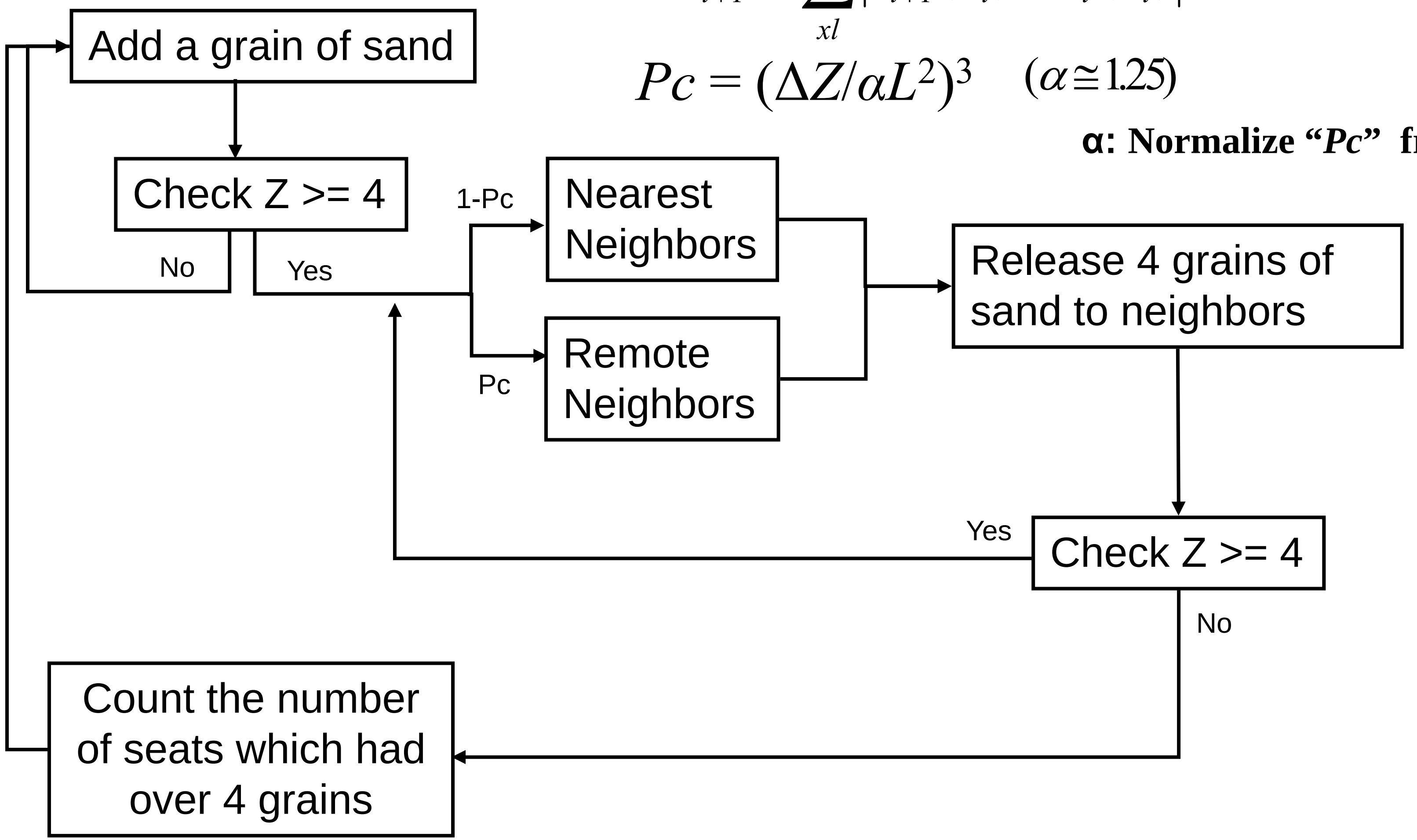



# Long-Range Connective Sandpile (LRCS) Model Flow Chart

$$\Delta Z_{i+1} = \sum_{xl} \left| z_{i+1}(x_l) - z_i(x_l) \right|$$

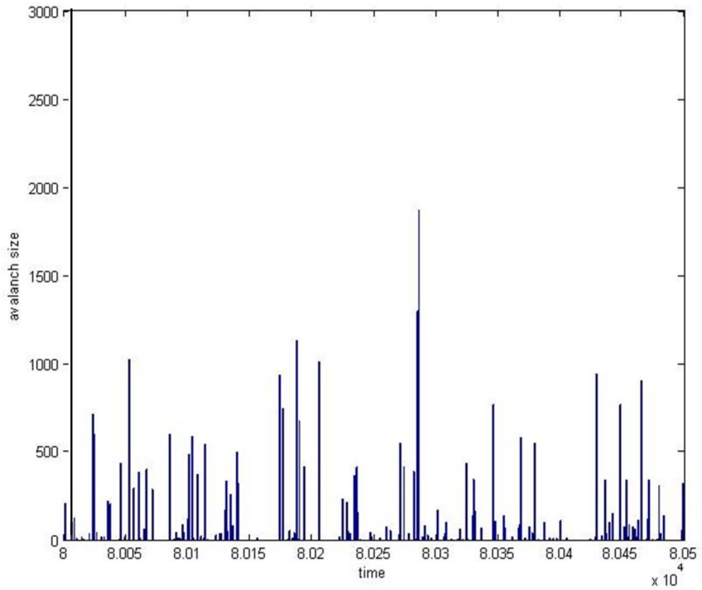
$$Pc = (\Delta Z / \alpha L^2)^3 \quad (\alpha \cong 1.25)$$

**α: Normalize "*Pc*" from 0~1**

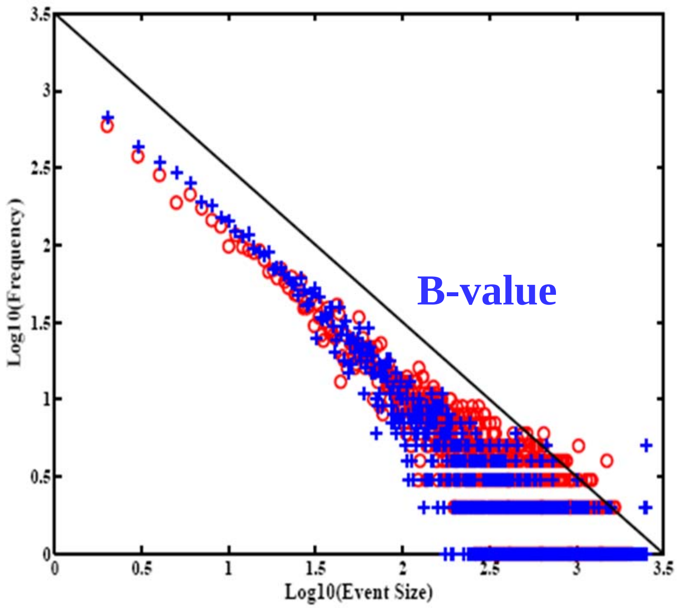


# Numerical (LRCS) Sandpile Model

## Event time series



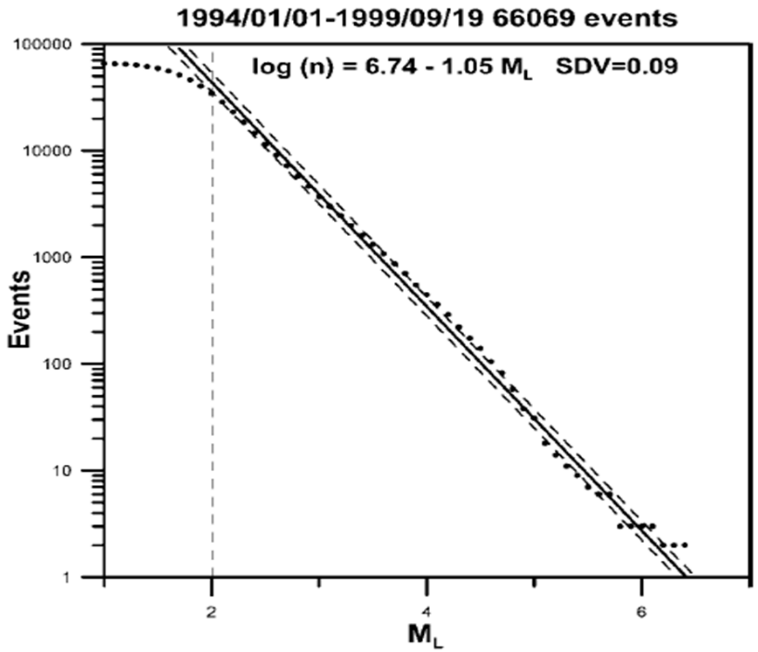

## Power-law frequency-size distributions



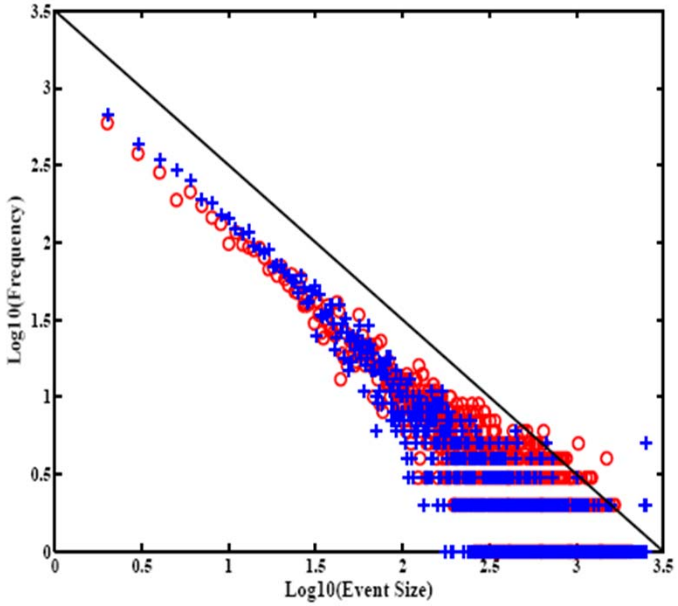

BTW sandpile model (red circles)
LRCS model (blue crosses)

# Numerical Sandpile Model (LRCS)

## Gutenberg-Richter law




## Power-law frequency-size distribution

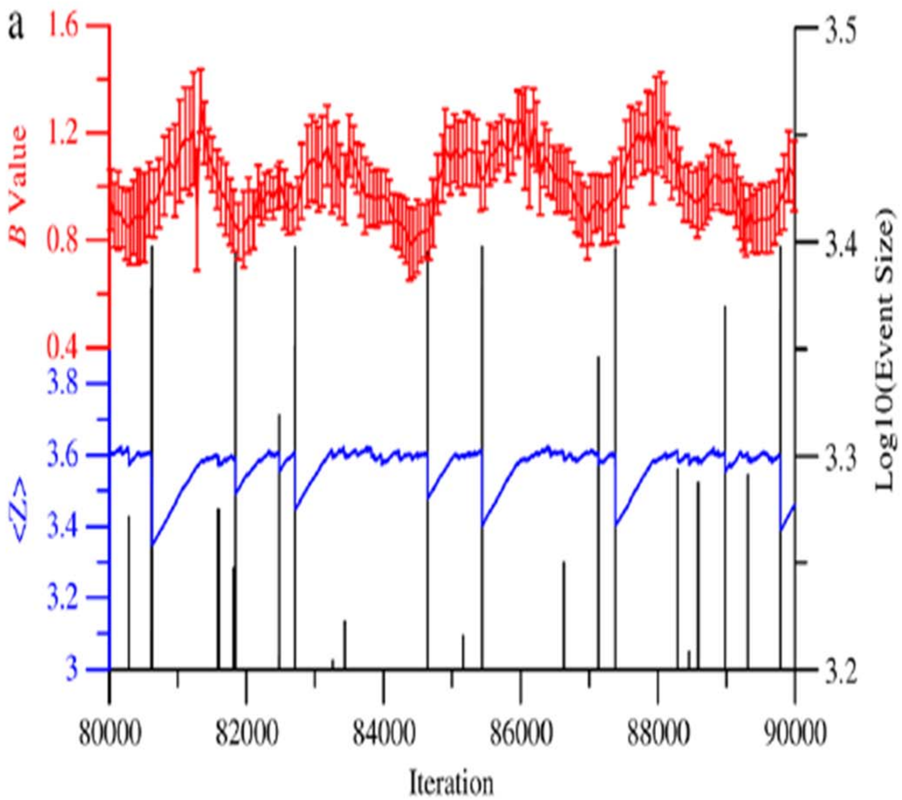



BTW sandpile model (red circles)

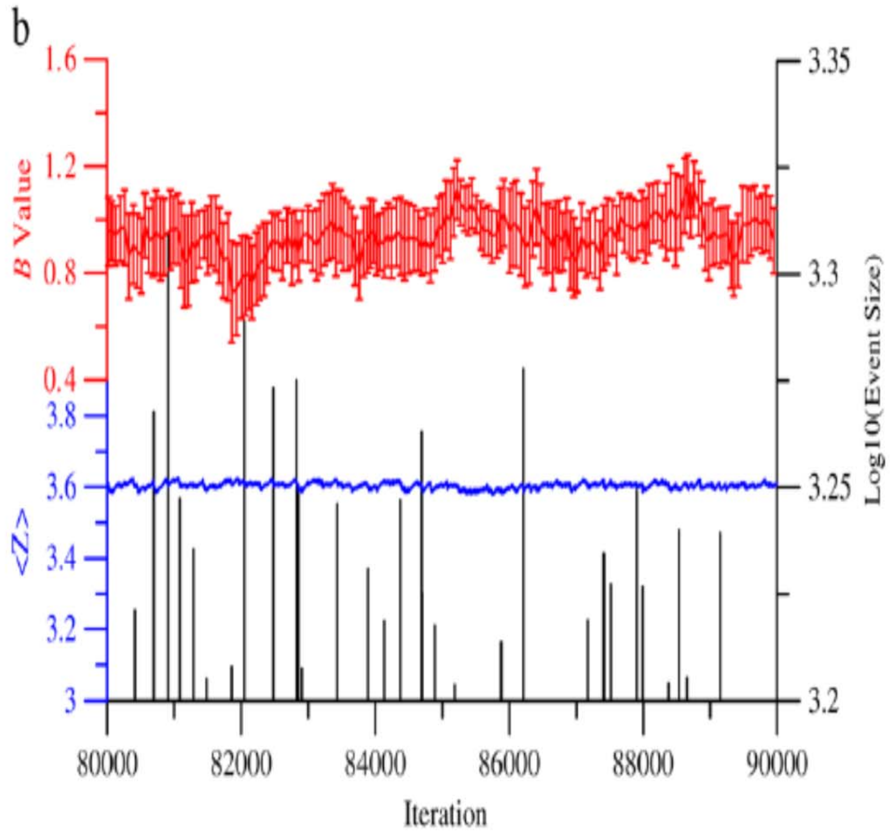
LRCS model (blue crosses)

# Intermittent Criticality

## LRCS model




## BTW sandpile model




[ Lee et al., 2008 *Physica A*]

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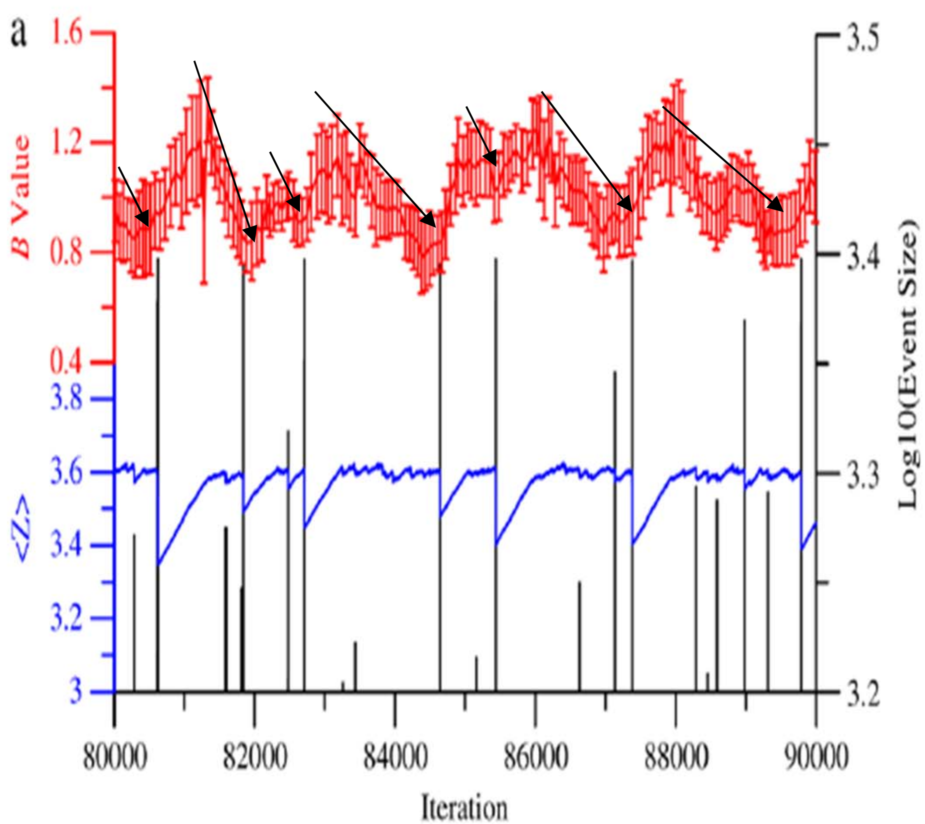
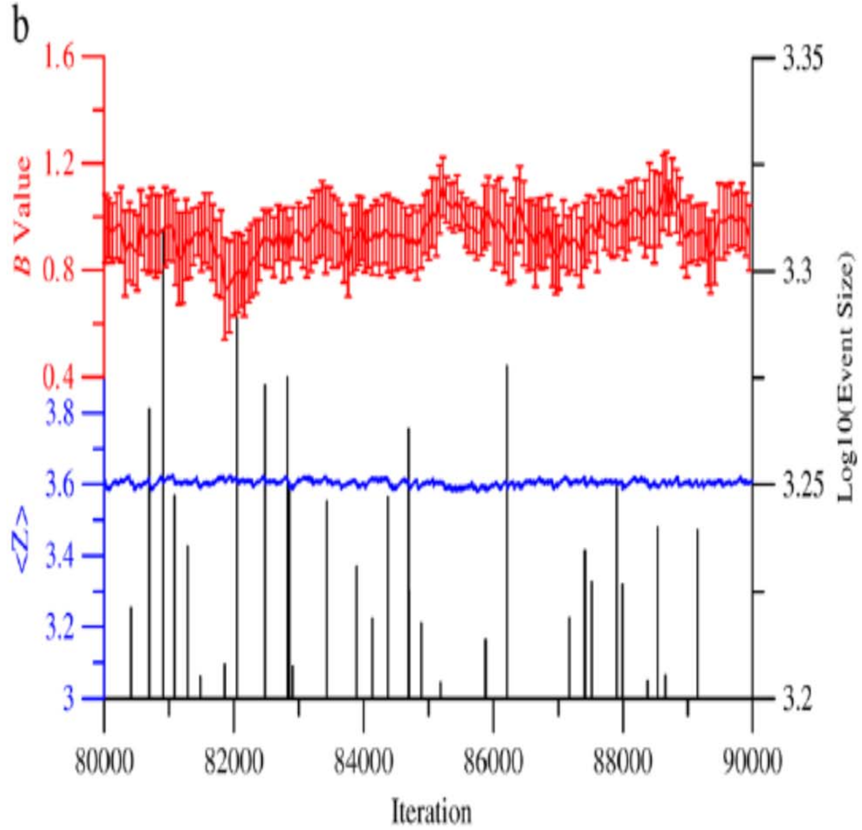
# B-value decay

## LRCS model




| Big event number | Event with *b* decay | Probability |
|---|---|---|
| 24 | 22 | 90% |

## BTW sandpile model

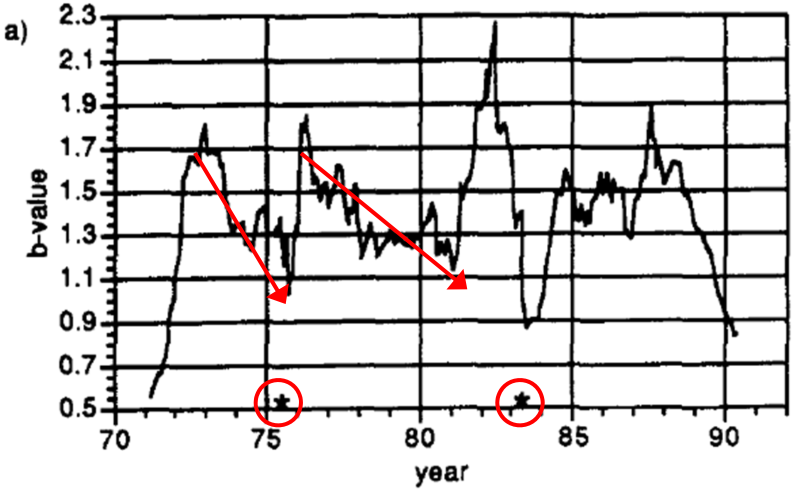



[ Lee et al., 2009 *GRL*]

# *b* Value Decay

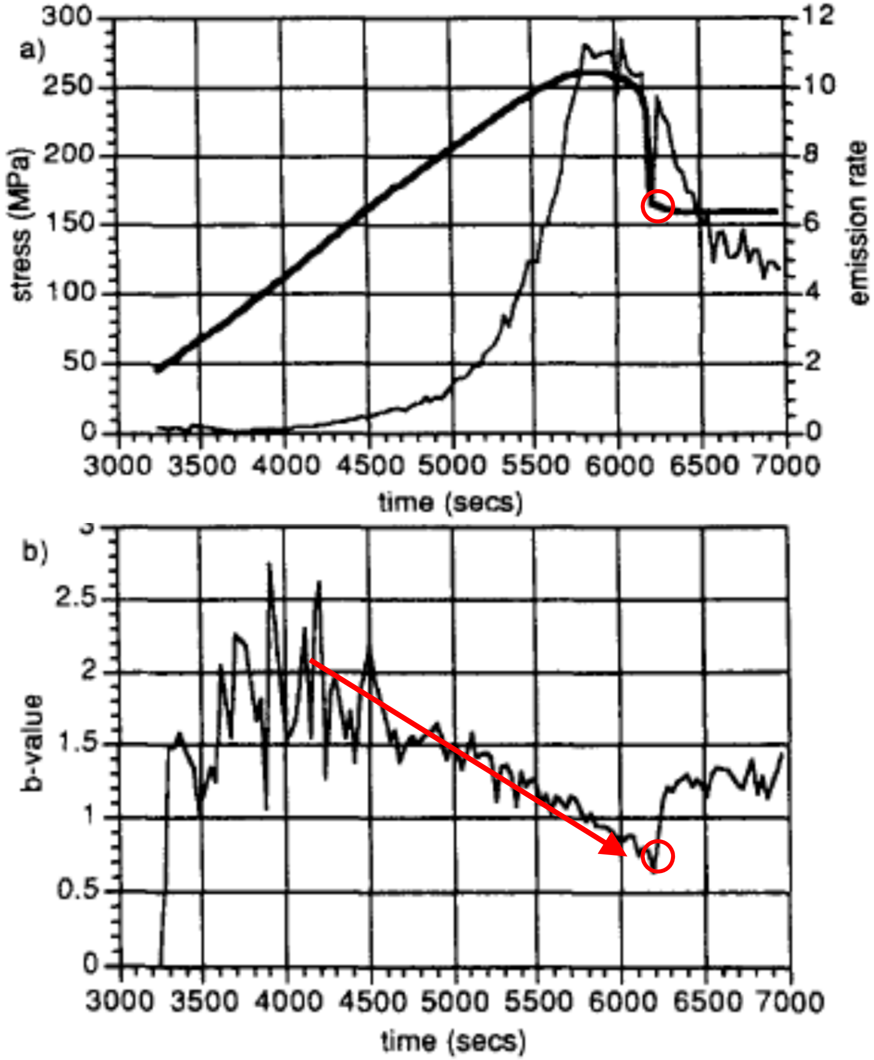
Jeremy Henderson
and Ian Main,1992

Parkfield



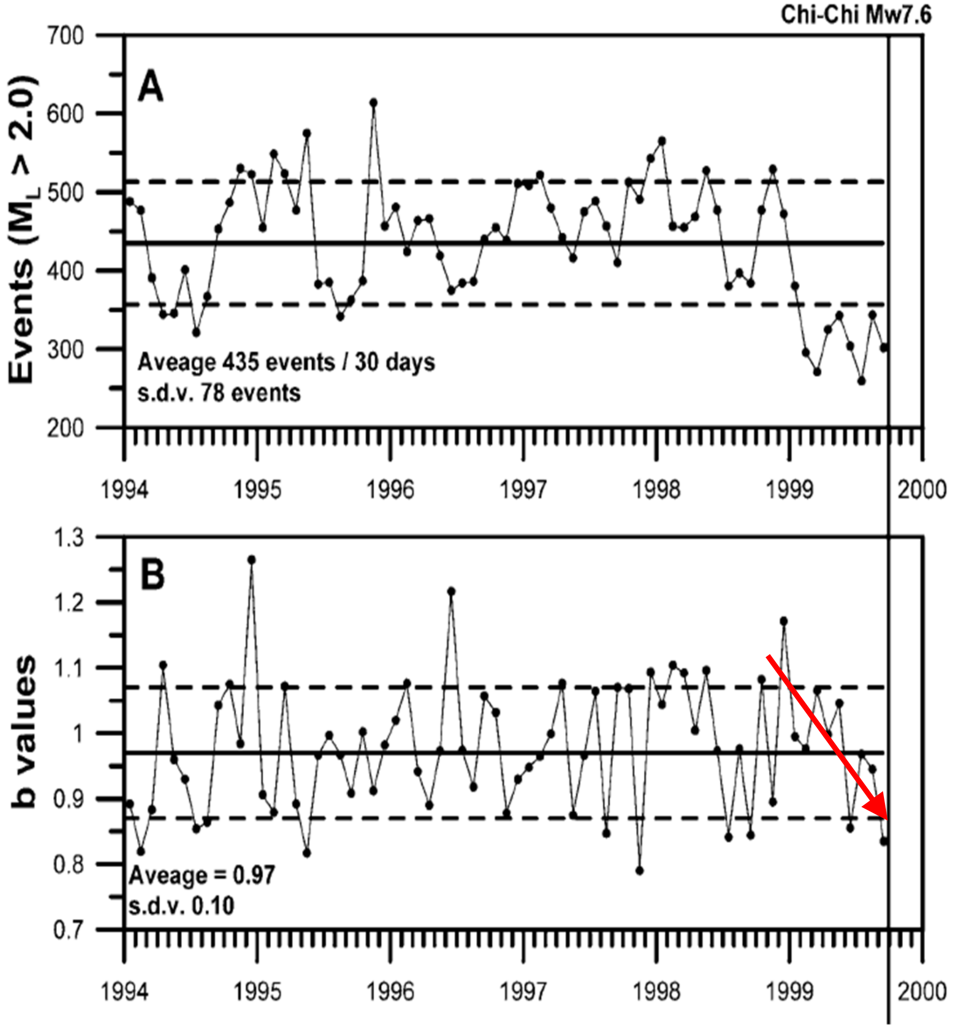

Middle Mountain area in 1975, M=4.5

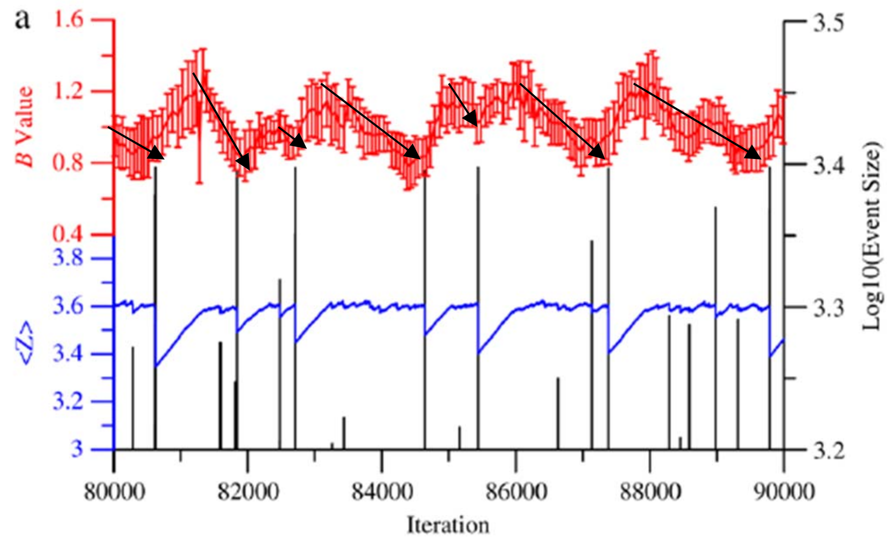
Coalinga in 1983, M=6.7

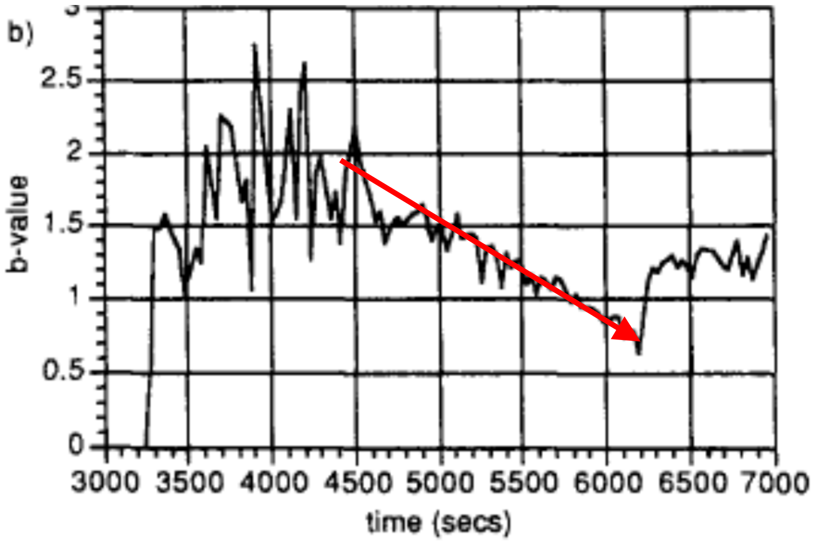



Digitally recorded acoustic emission in the laboratory.

# *b* Value Decay



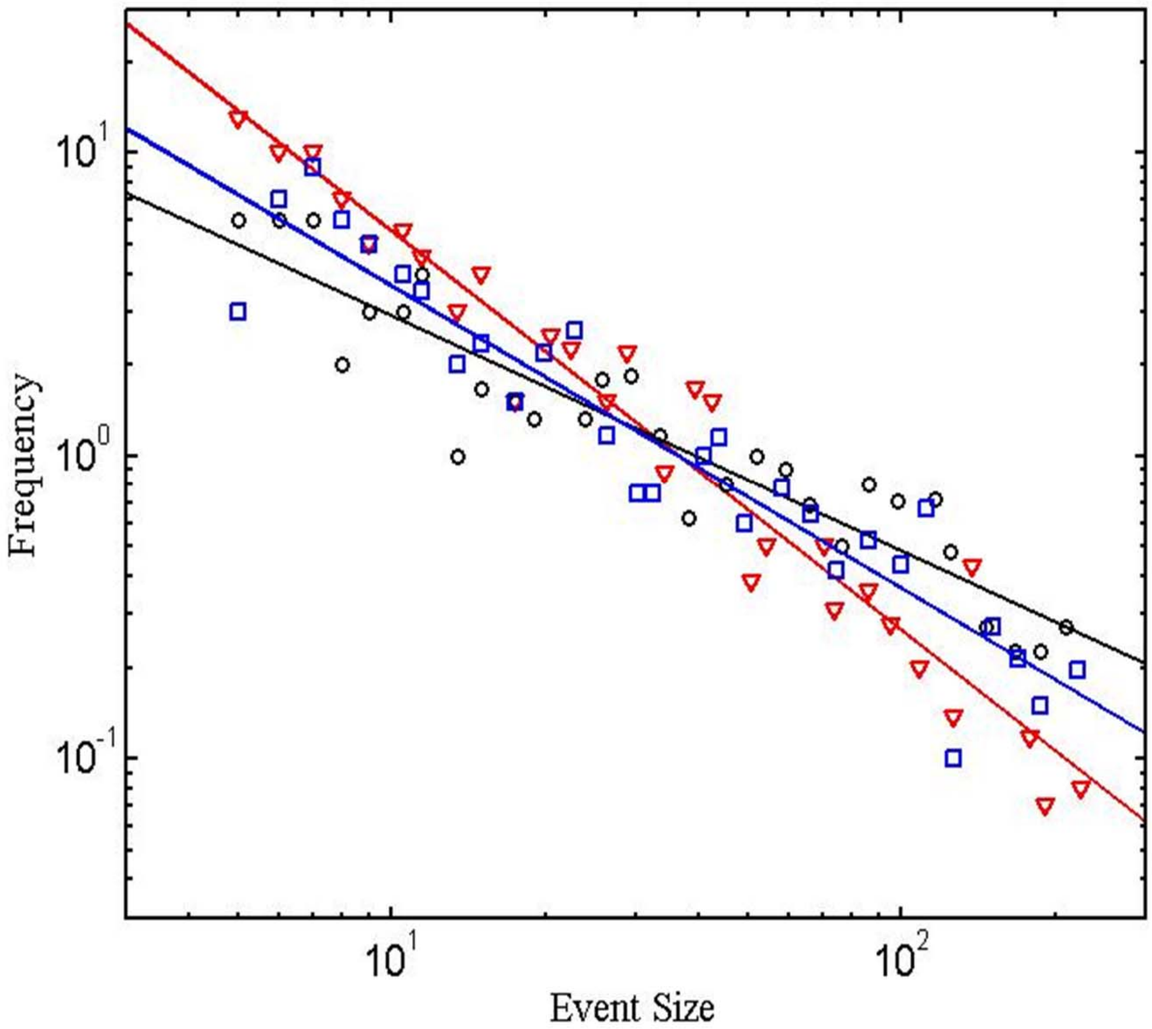




**Wu and Chiao, 2006**

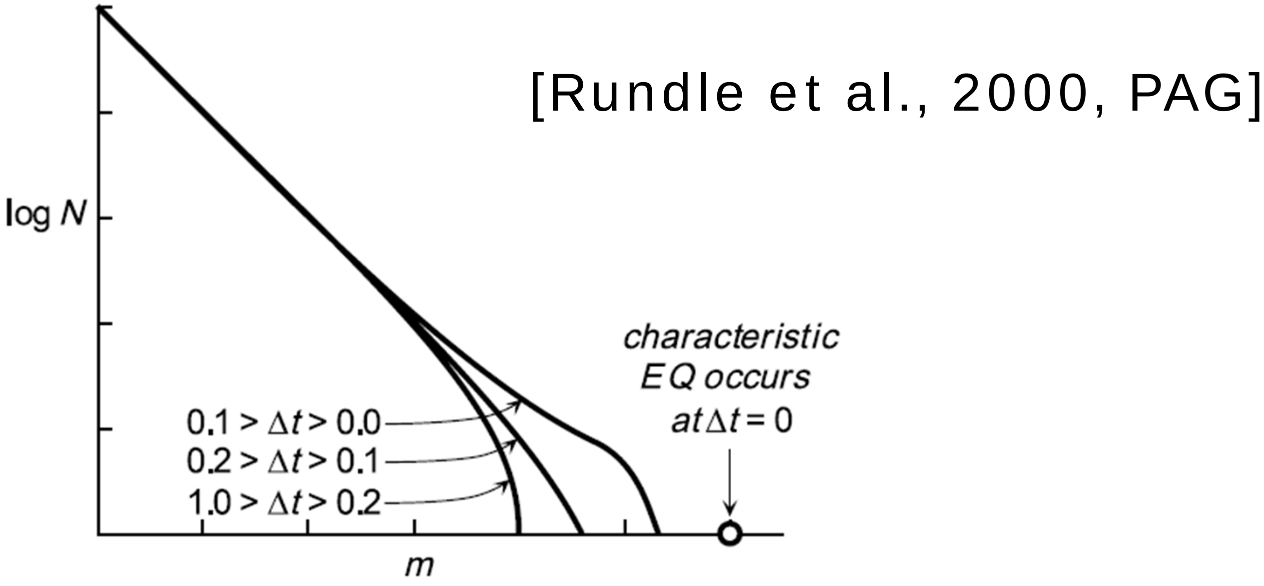
Digitally recorded acoustic emission in the laboratory.

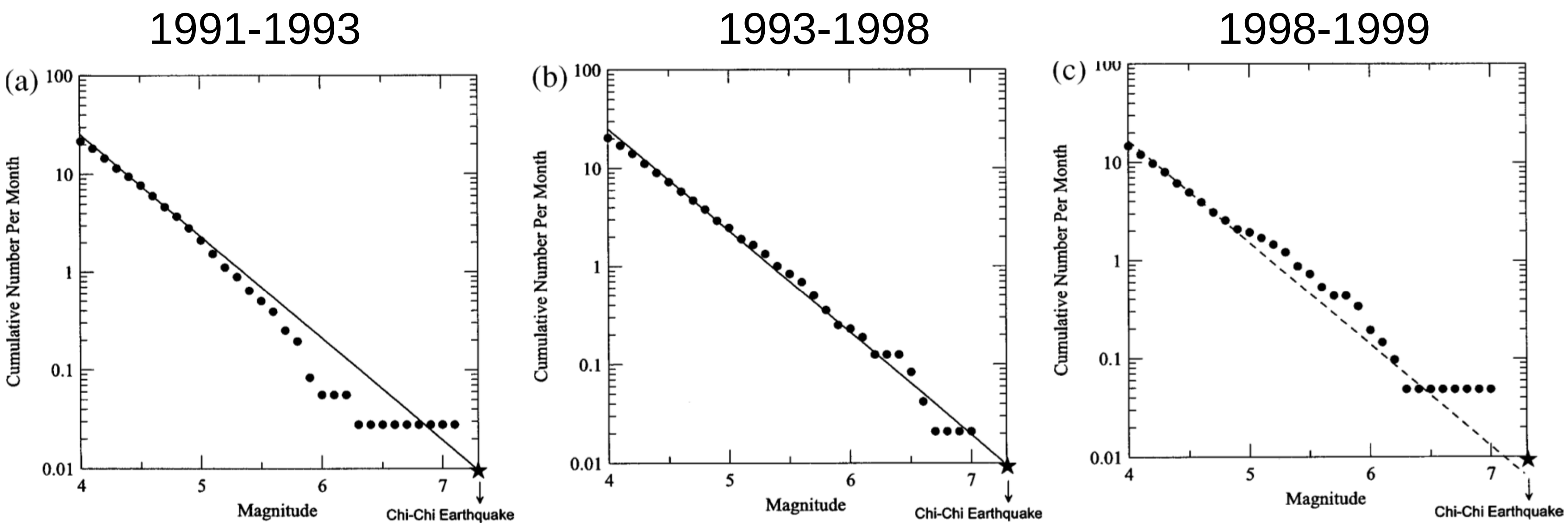
# Slope of Frequency and Event Size




- ▽ **High (>1.3, red Inverted triangles)**
- □ **Middle (~1.0, blue squares)**
- ○ **Low (<0.8, black circles)**

# Self-Organizing Spinodal (SOS) Model




**Chen, 2003 Geophys. J. Int.**

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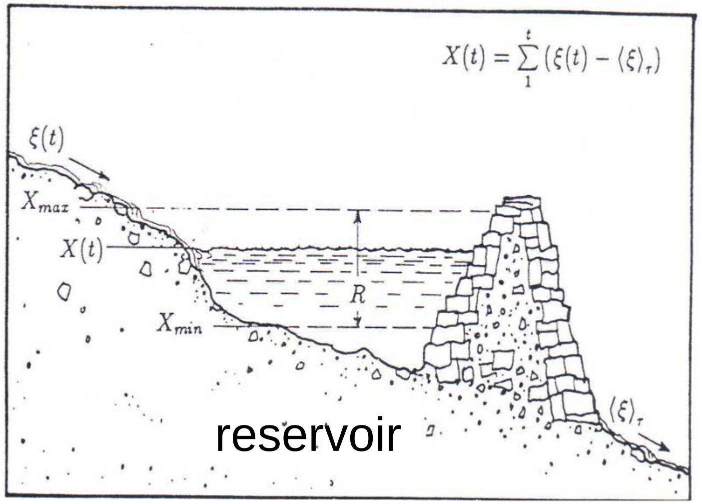
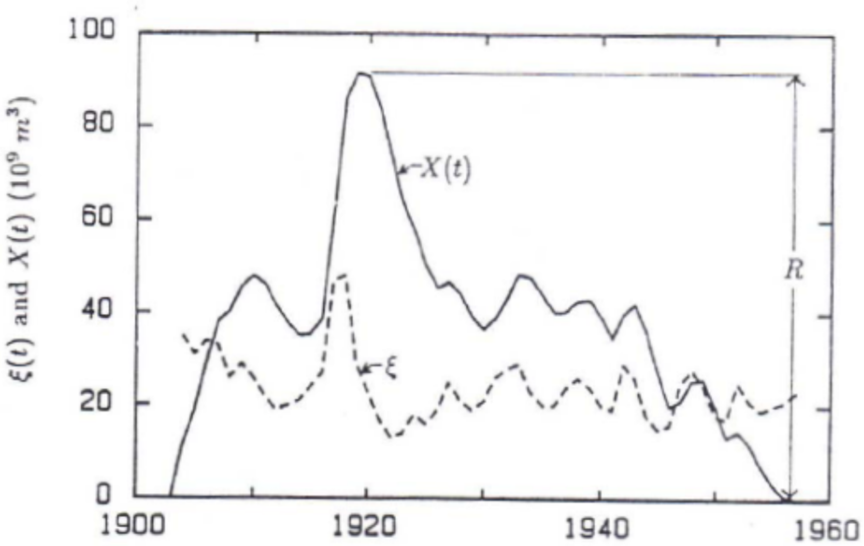
# Rescaled Range Statistical Analysis

## [Hurst 1951]




## R/S analysis :

$$X(t,\tau) = \sum_{u=0}^{t} \left[ q(t) - \langle q \rangle_\tau \right] \Delta t$$

$$\langle q \rangle_\tau = \frac{1}{\tau} \sum_{t=1}^{\tau} q(t)$$

$$R(\tau) = \max_{1 \le t \le \tau} X(t,\tau) - \min_{1 \le t \le \tau} X(t,\tau)$$

$$R(\tau)/S(\tau) \propto \tau^{H}$$

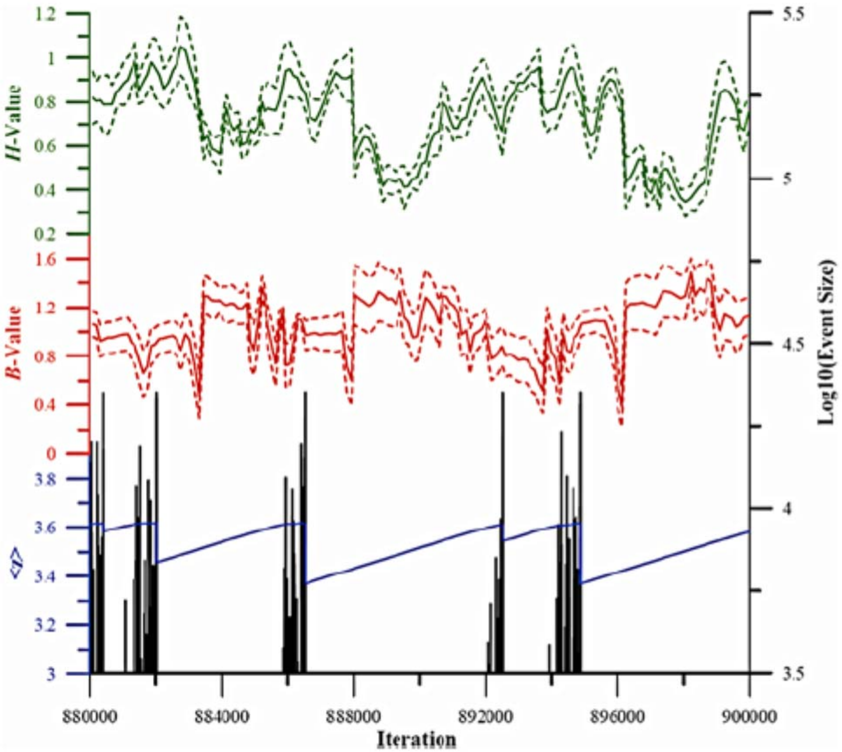
**0.5<H： persistent**

**0<H<0.5： anti-persistent**

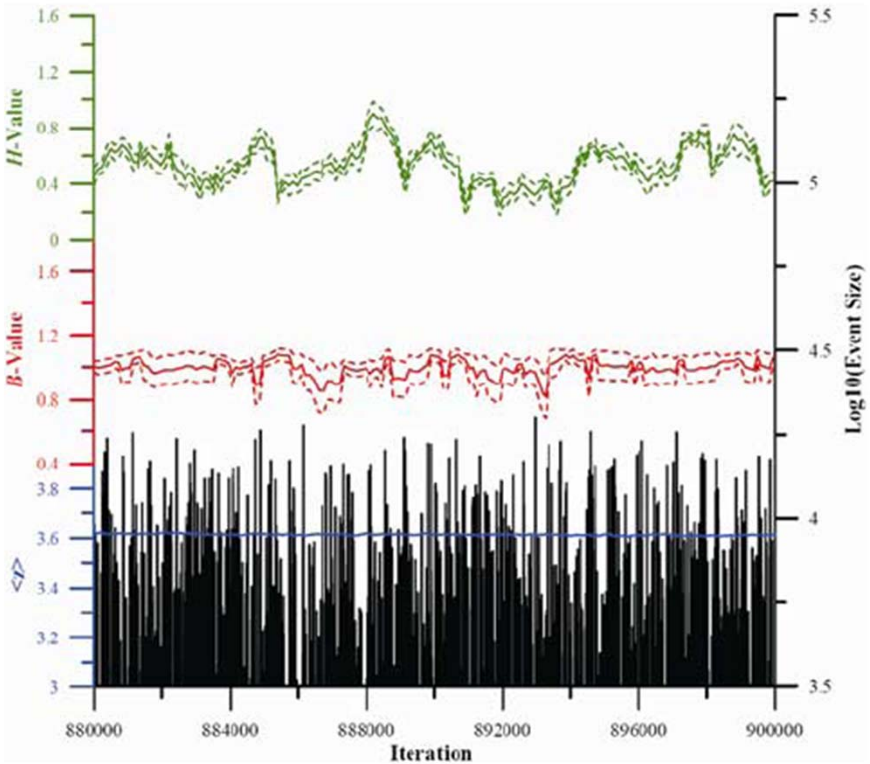
**H=0.5： random**

# *B*-Value vs. Hurst Exponent

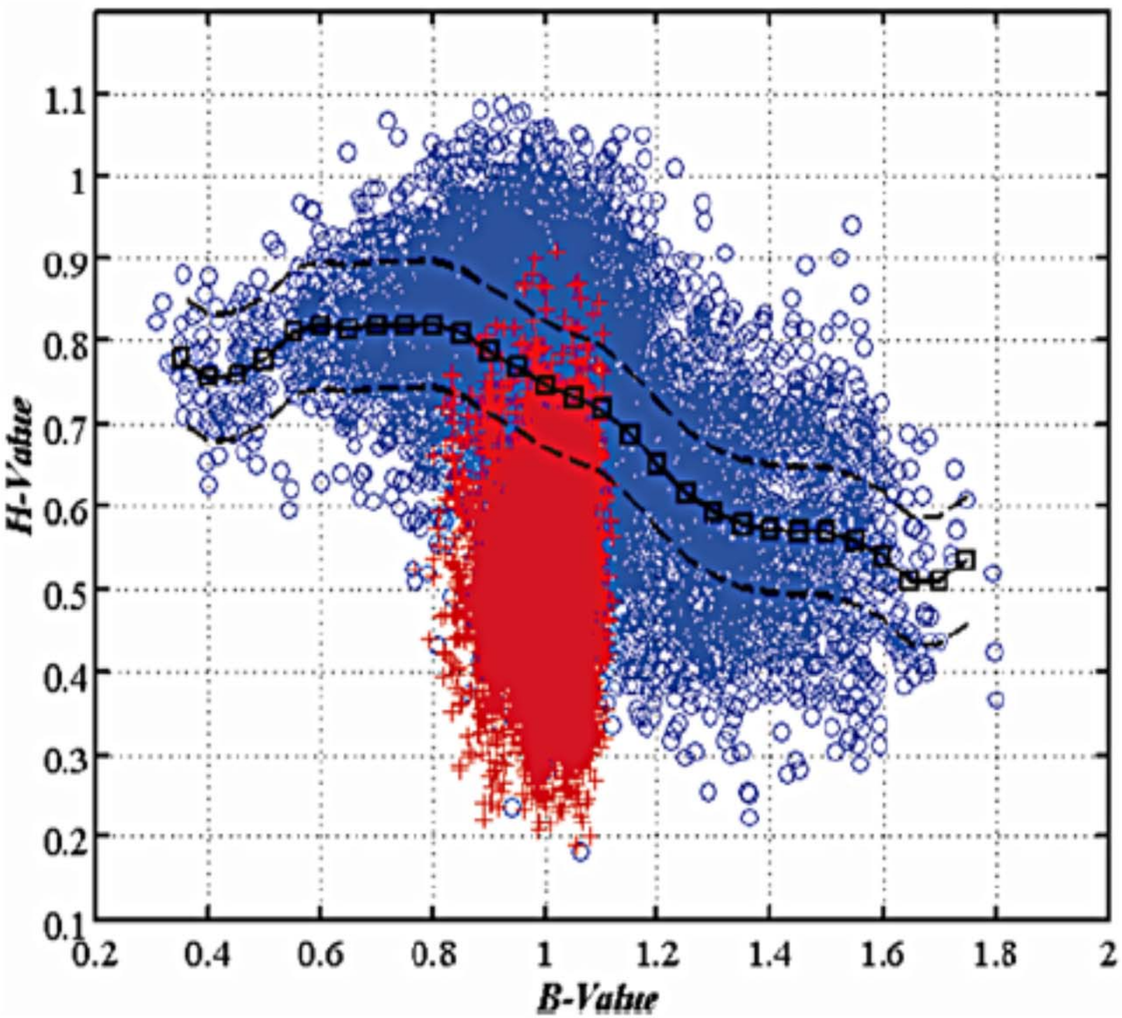
## LRCS model




## BTW sandpile model



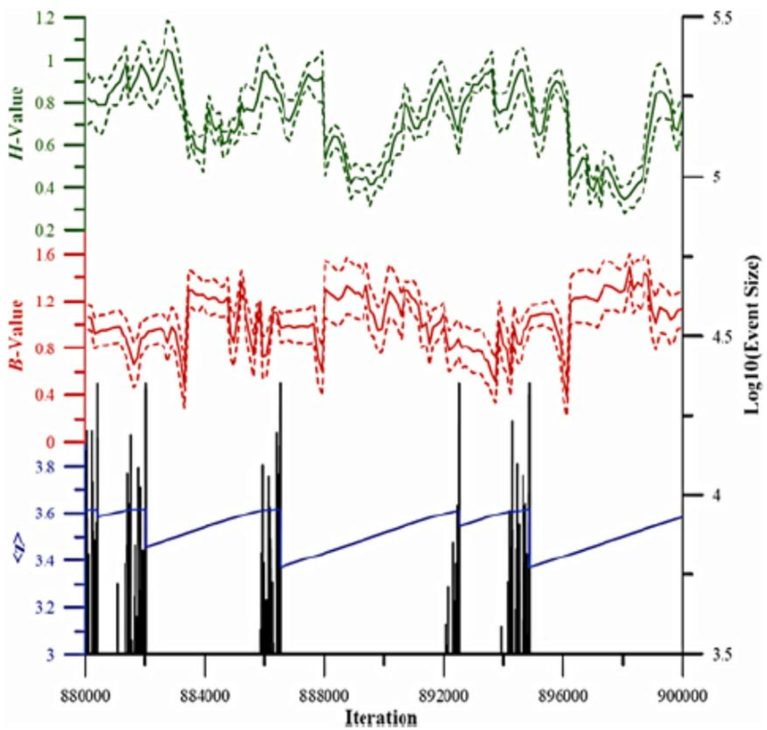
# *B*-Value vs. Hurst Exponent



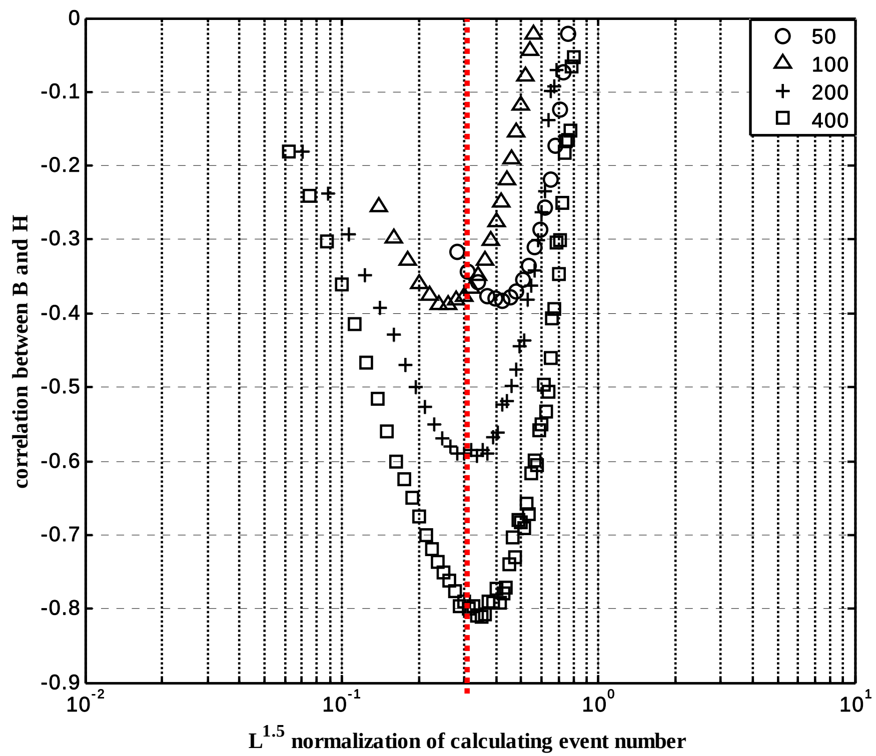

[Lee et al., 2009, *Geophys. Res. Lett.*]

# Time Window Selection

## B-value vs. H-value

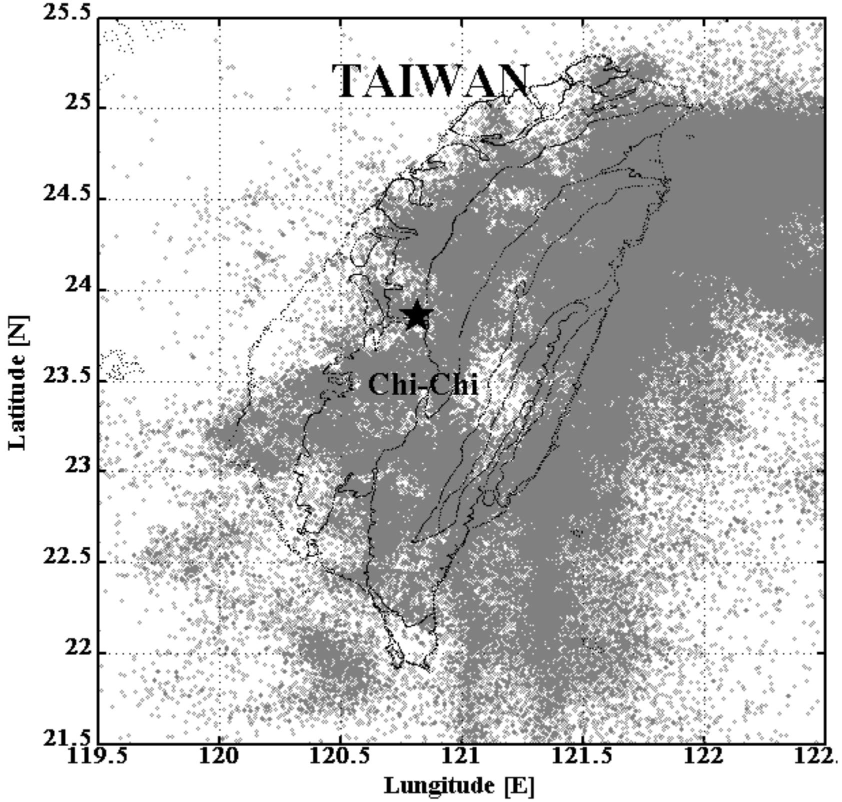



## Correlation Coefficient of B-value and H-value

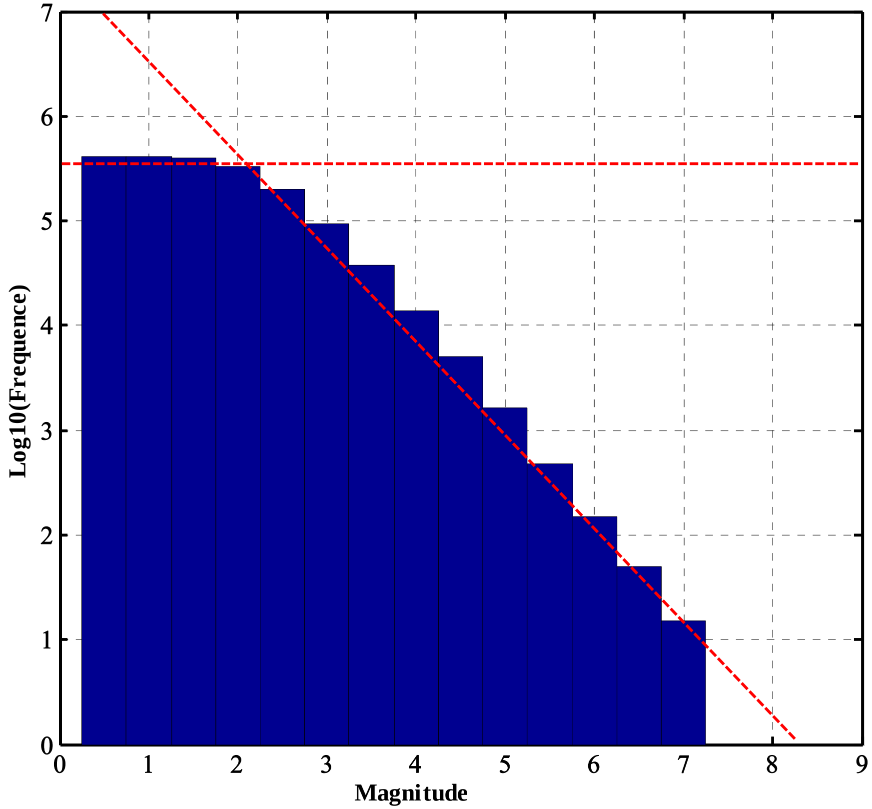


# Taiwan Earthquake Catalog

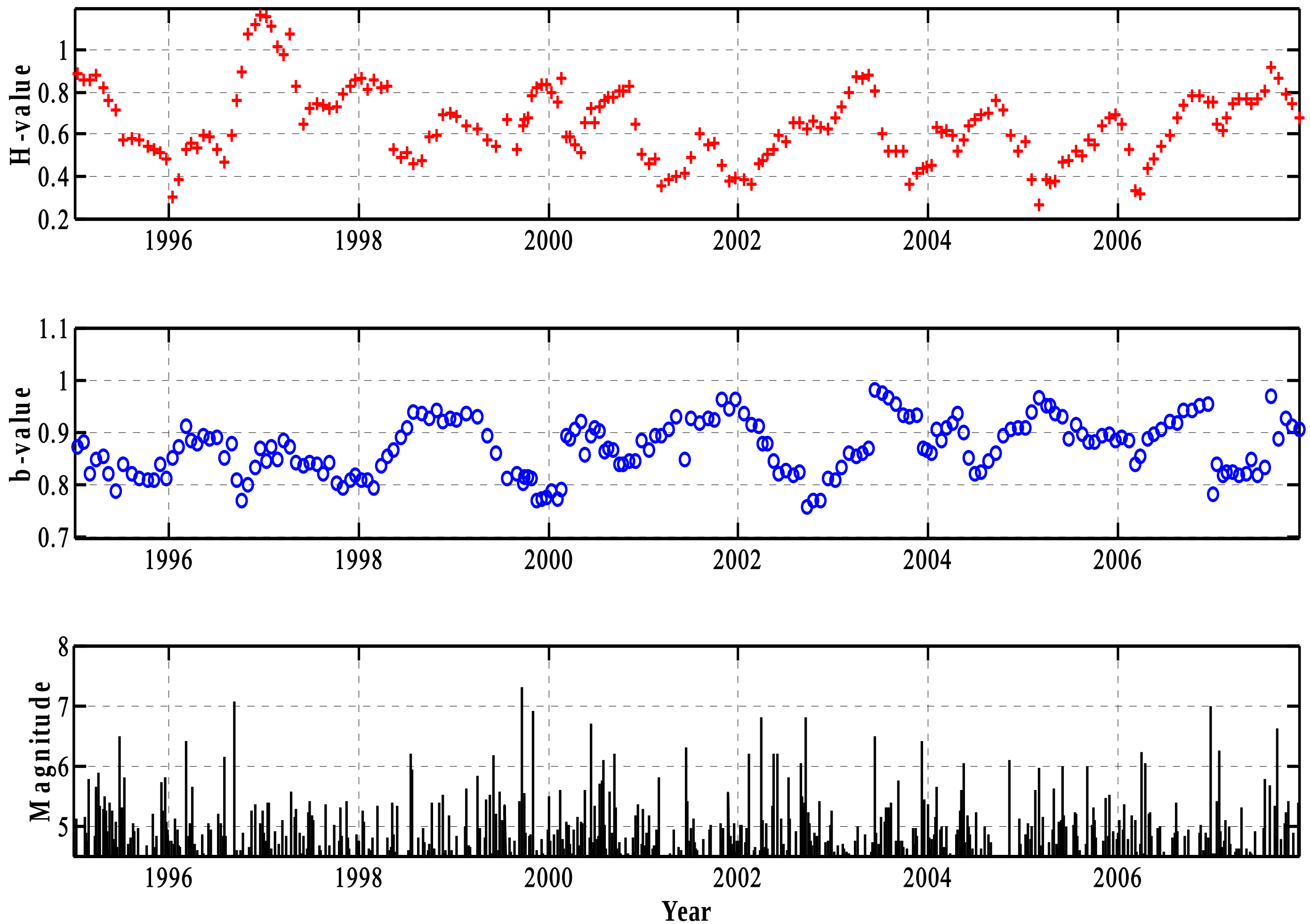
## 1995-2007




## Gutenberg-Richter law

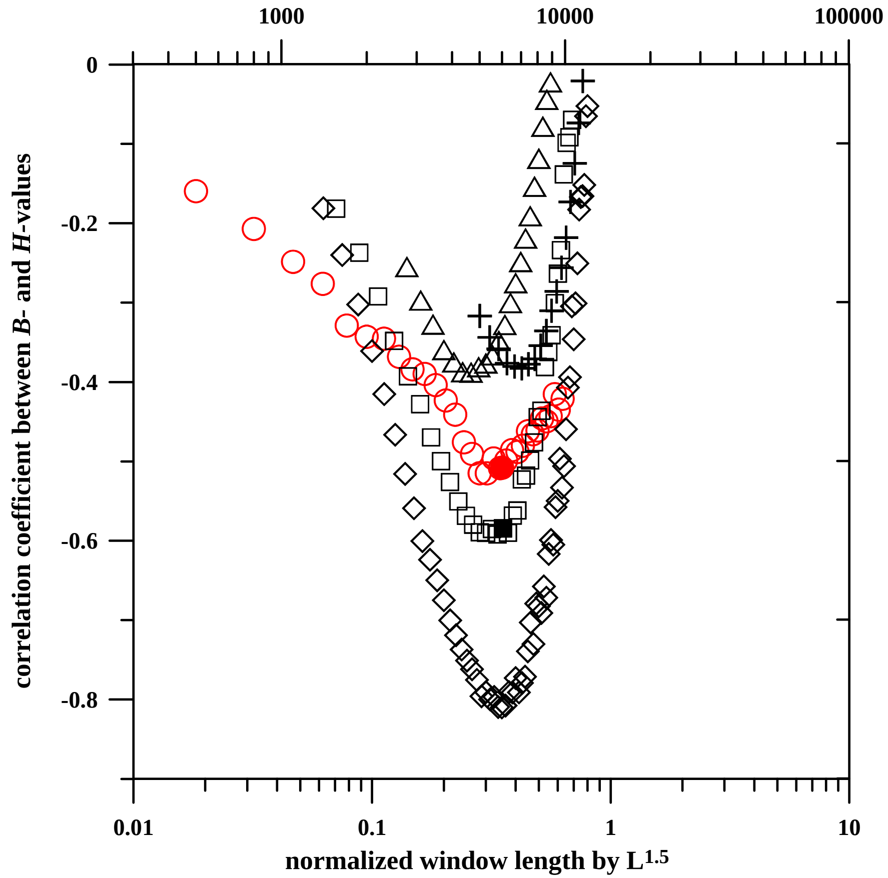


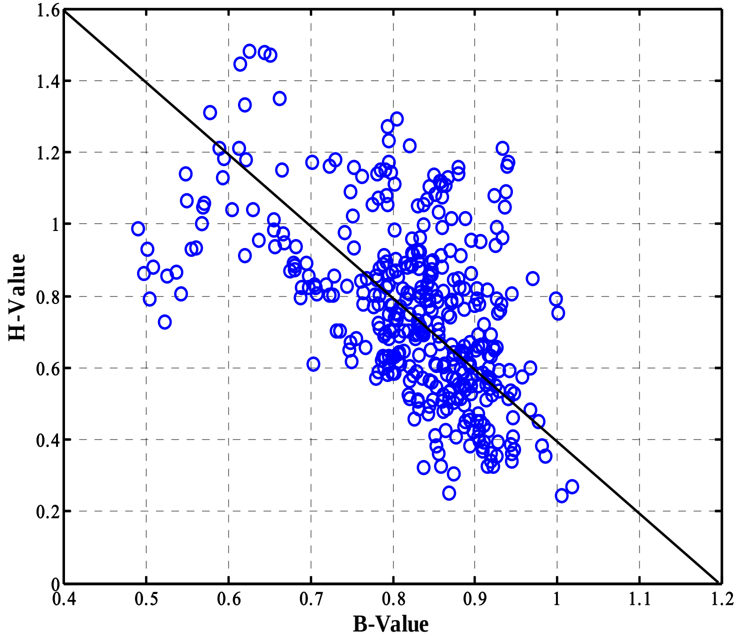
# *b*-values vs. *H*-values for Taiwan Earthquake



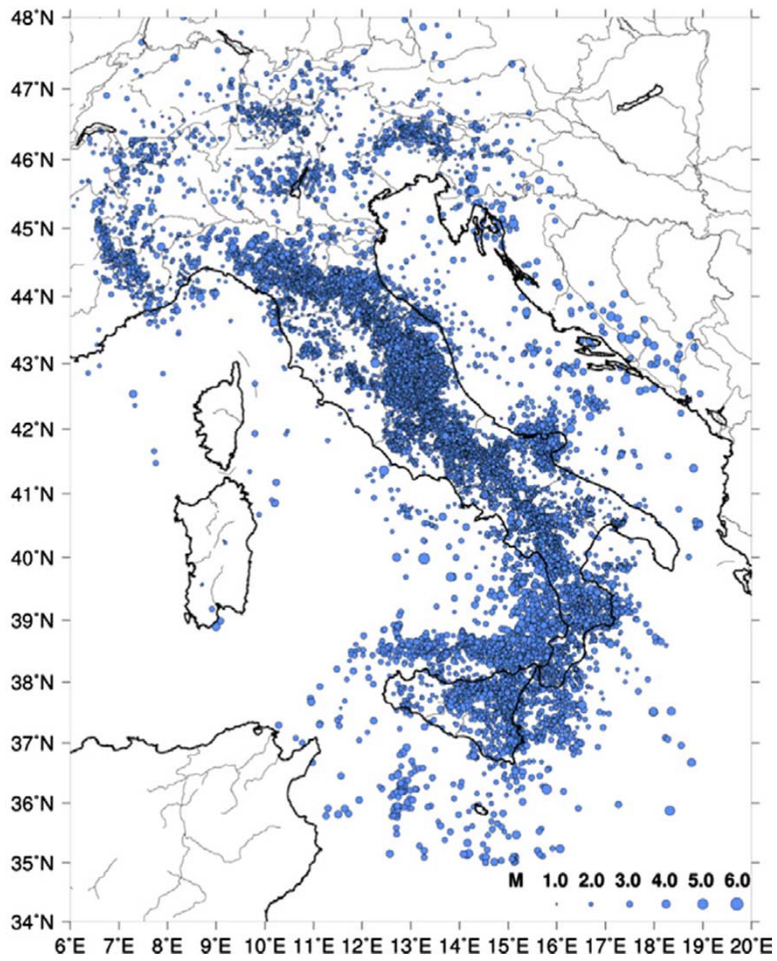
# Time Window Selection

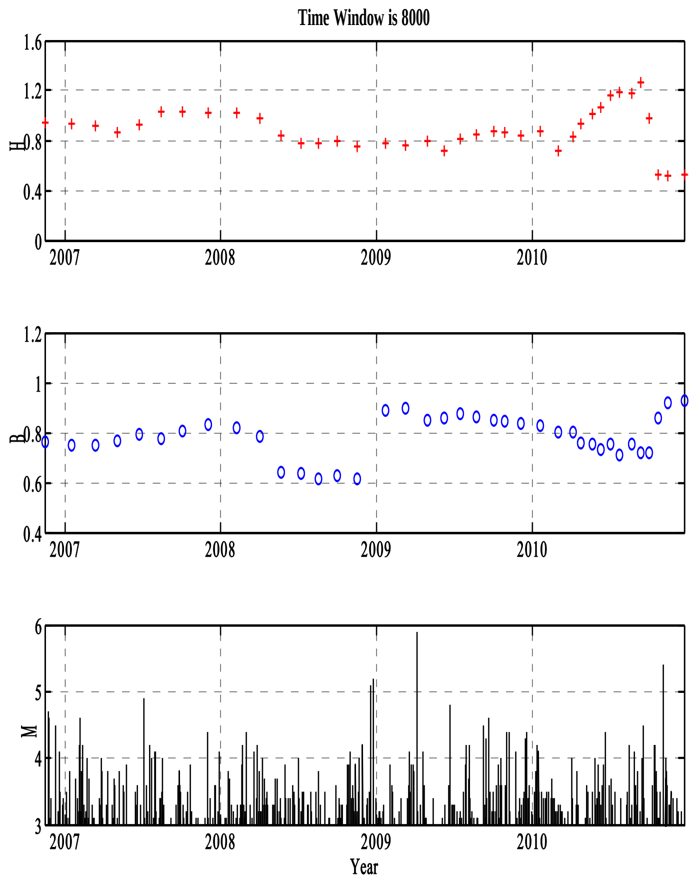
•Correlation coefficient between *b*- and H-values for De-cluster Earthquake data



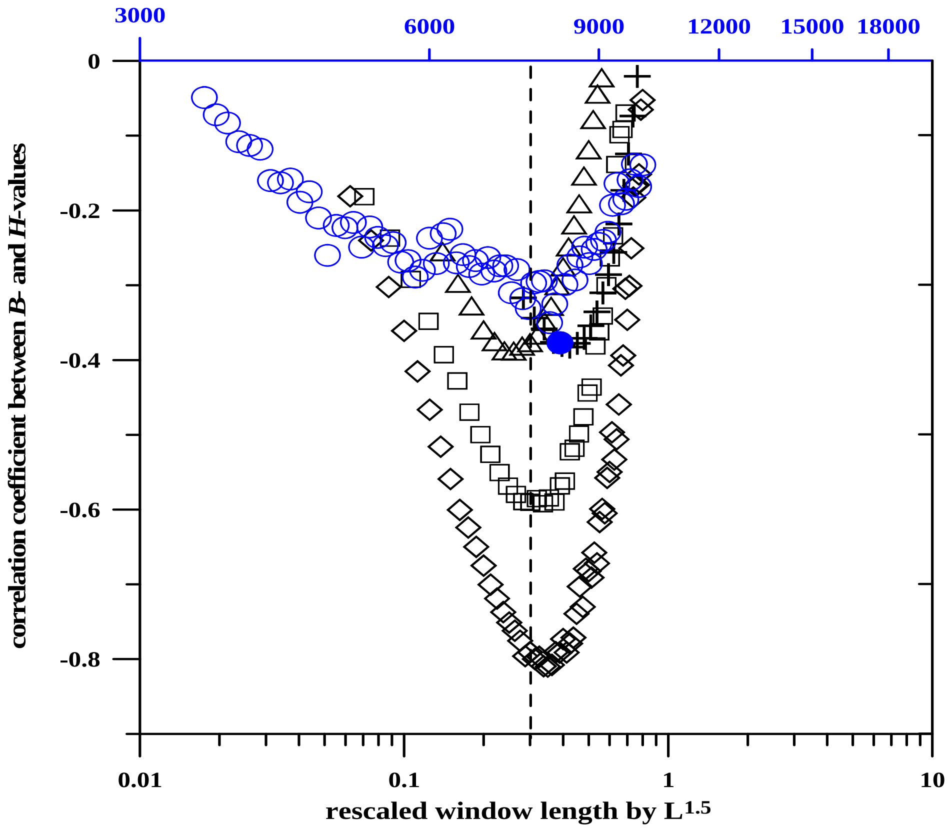
# Italy Earthquake

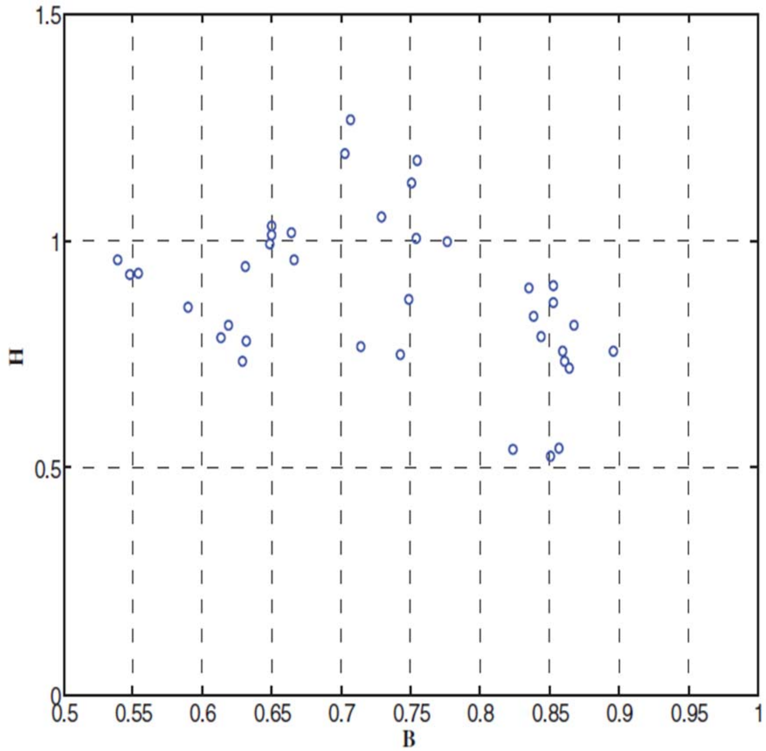


# Italy Earthquake

- Negative correlation between *b*- and H-value



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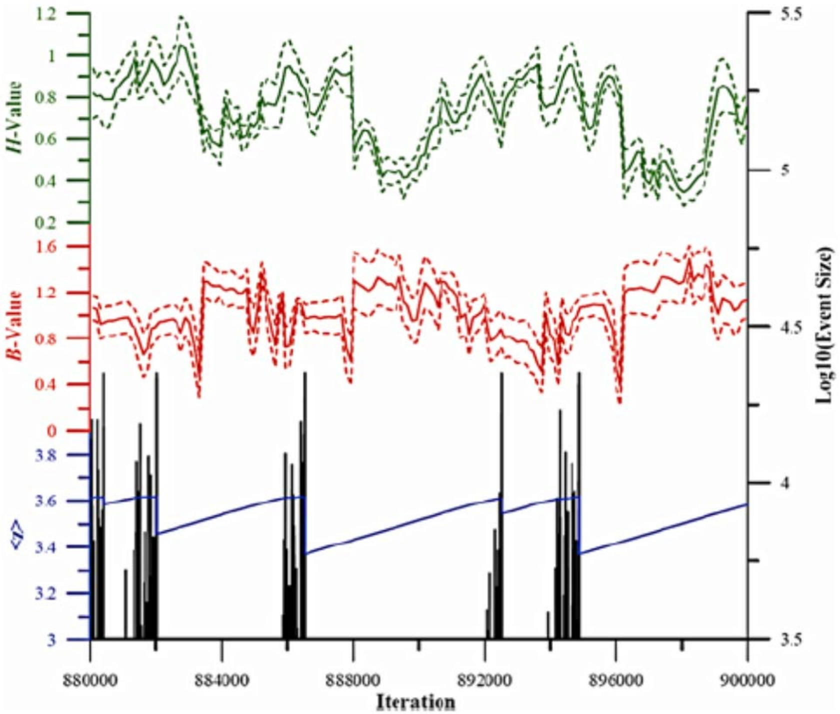
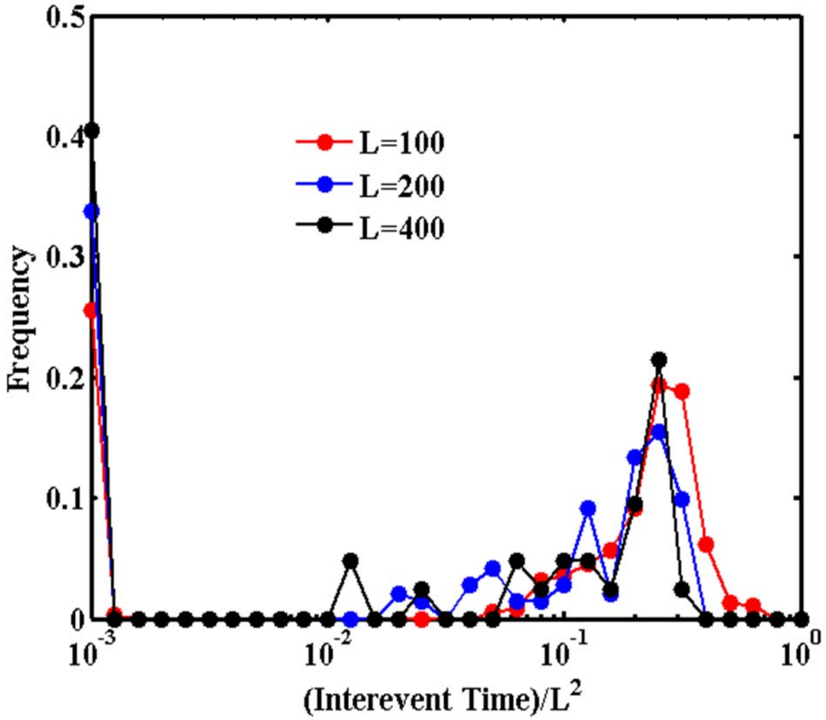
# Quasi-Periodicity

**Periodicity of B- and H-value**

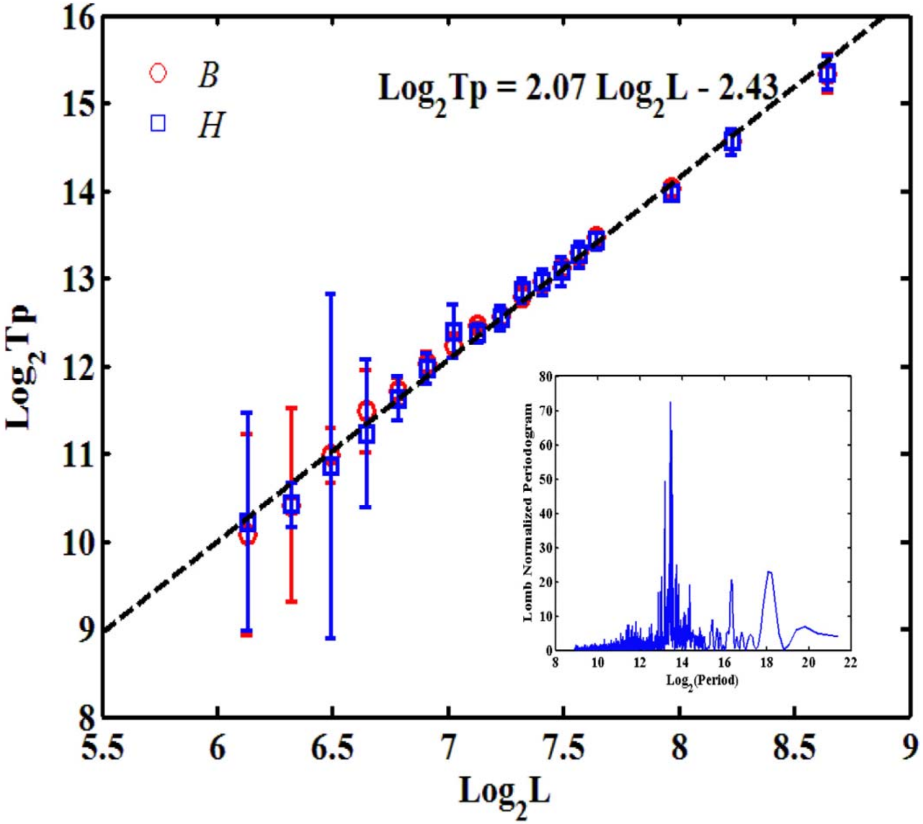



**Periodicity of big event**



# Quasi-Periodicity

- Periodicity of B- and H-value vs. system size




$$P(s) = s^{-\tau} G(s / L^D),$$

$$\Delta P = \int_{aL^2}^{L^2} P(s) ds$$

$$\Delta P = \int_{aL^2}^{L^2} s^{-\tau} G(s / L^D) ds$$

$$= L^{D(1-\tau)} \int_{aL^{2-D}}^{L^{2-D}} (s / L^D)^{-\tau} G(s / L^D) d(s / L^D)$$

$$\cong L^{D(1-\tau)} \int_a^1 u^{-\tau} G(u) du$$

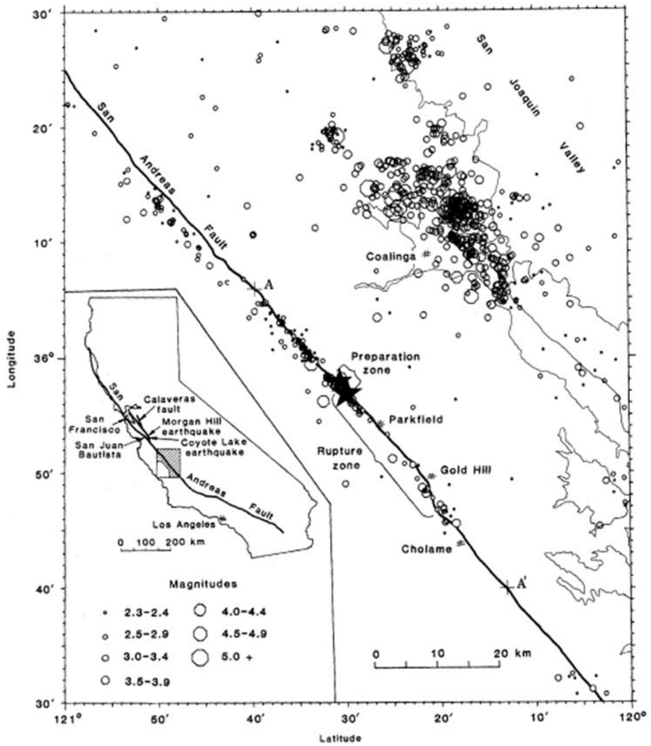
$$\boxed{Tp = t \times N_T = t \times N_a / \Delta P \propto L^m / L^{D(1-\tau)} = L^{D(\tau-1)+m}}$$

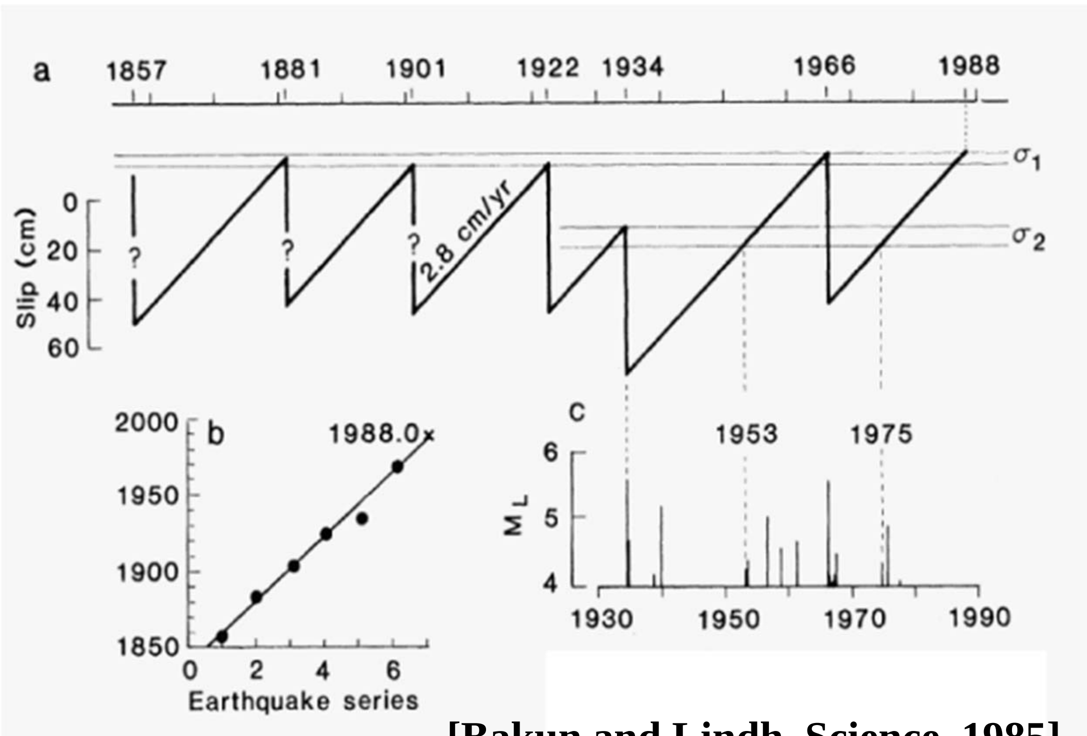
# Period of Large Event vs. System Size

•Parkfield, California, earthquake prediction experience

**T = 1857, 1881, 1901, 1922, 1934 and1966;**
**$T_R$ = 22 years; M =~6.0 ;**

**Parkfield**




[Bakun and Lindh, Science, 1985]

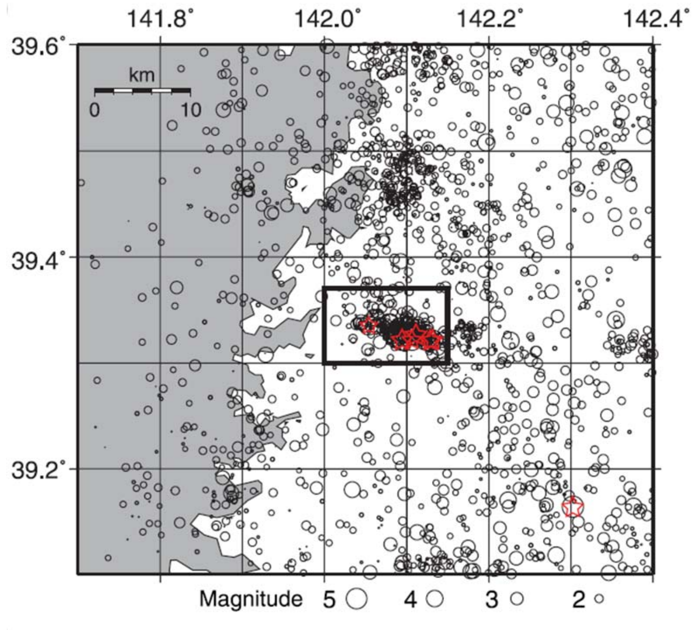
# Period of Large Event vs. System Size

- Kamaishi, NE Japan earthquake cycle slate

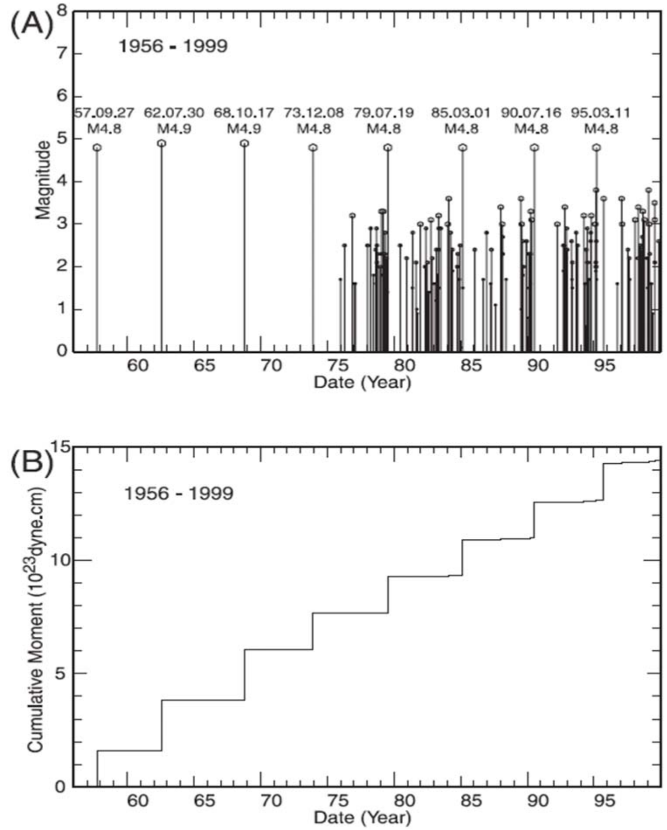
**T = 1957, 1962, 1968, 1973, 1979, 1985, 1990 and 1995 ;**

**$T_R$ = 5.35 years; M = 4.8 ;**

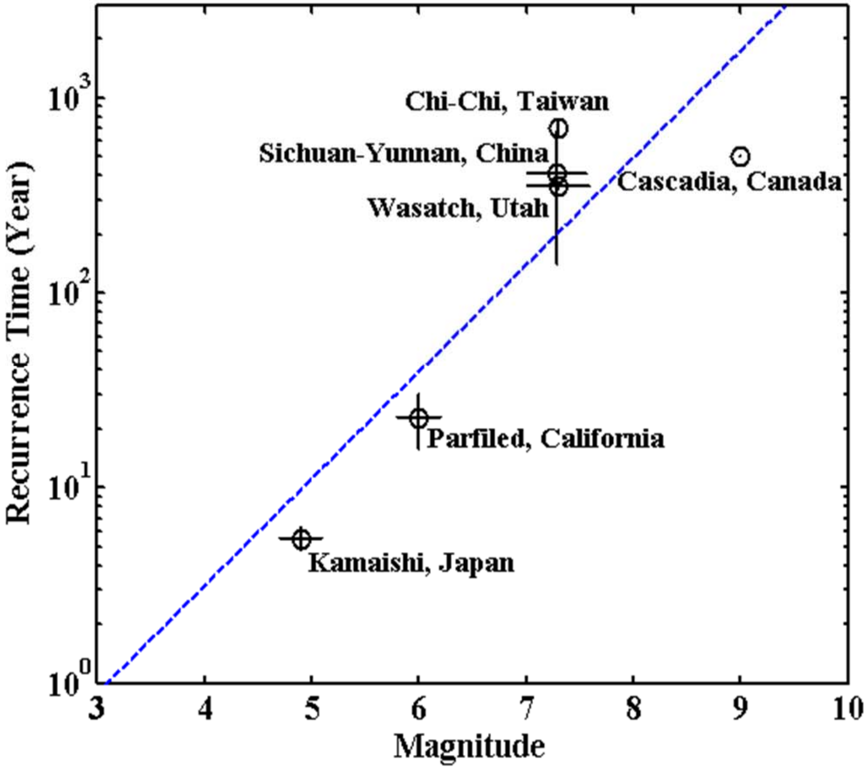
**NE Japan**

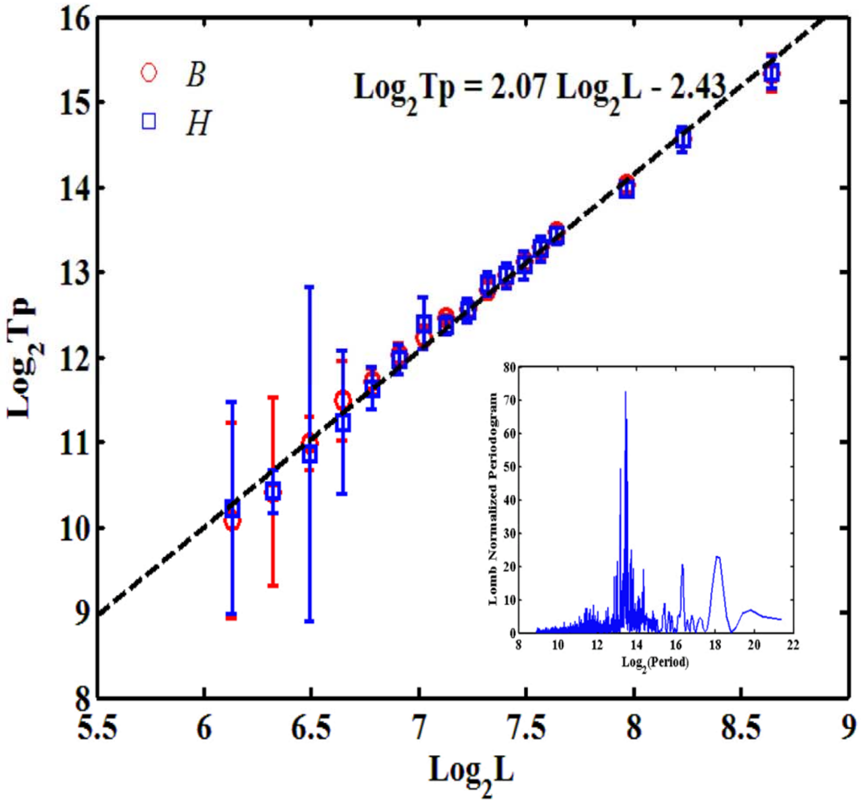



[Matsuzawa et al. GRL, 2002]



# Characteristic Earthquake Periodicity



# Conclusion I

- Alternative sandpile model variant through adaptable long-range connections.
- LRCS model
  - Intermittent criticality
  - B-value decay
  - Negative correlation between *B*- and *H*-values
  - Quasi-periodicity

# Conclusion II

- Using LRCS model we suggest that the *b*-value reduction before big earthquake might be caused by changes in criticality.
- The statistical parameter Hurst exponent could be as another precursor index.
- The characteristic earthquake have quasi-periodicity behavior.
- Statistical physics methods are useful in understanding complex phenomena of seismicity system by a simple model and rules.

Thanks for your attention