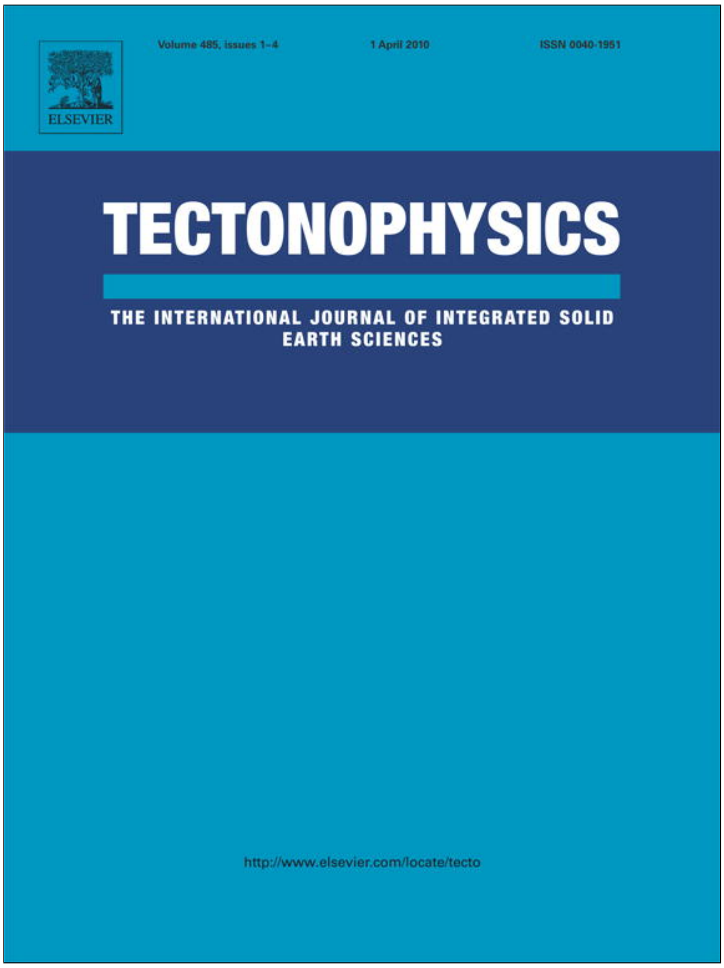
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The Weibull–log Weibull transition of interoccurrence time for synthetic and natural earthquakes

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ABSTRACT

We study the interoccurrence time distributions of events by analyzing synthetic catalogues and three natural catalogues of the Japan Meteorological Agency (JMA), the Southern California Earthquake Data Center (SCEDC) and the Taiwan Central Weather Bureau (TCWB). We find a universal feature, i.e. the Weibull–log Weibull transition, in the interoccurrence time statistics. This transition demonstrates that the interoccurrence time statistics of earthquakes possess the hybrid Weibull and log Weibull statistics. We further find that the crossover magnitude m_c^{**} from the superposition regime to the pure Weibull regime is averagely proportional to the plate velocity. In the end of this paper we summarize a region-independent relation, i.e. $m_c^{**}/m_{\max} = 0.54 \pm 0.06$, which represents a novel empirical relation related to the Weibull–log Weibull transition for earthquake processes.

1. Introduction

Statistical properties of time intervals between successive earthquakes (hereinafter the interoccurrence times and the recurrence times) have been frequently studied in order to predict when the next big earthquake will happen. Interoccurrence times and recurrence times mean the time intervals between the events on all faults in a region and on a single fault/segment, respectively. Previous studies (Utsu, 1984; Madhava Rao and Kaila, 1986; Papadopoulos, 1987; Papazachos et al., 1987; Dionysiou and Papadopoulos, 1992; Wang and Kuo, 1998; Bak et al., 2002; Matthews et al., 2002; Corral, 2004; Shcherbakov et al., 2005; Abaimov et al., 2008; Enescu et al., 2008) have been mainly focused on the determination of the underlying probability distribution and the presentation of the scaling law. For instance, the Weibull distribution (Abaimov et al., 2008), the exponential distribution (Enescu et al., 2008), the Brownian passage time distribution (Matthews et al., 2002), the gamma distribution (Wang and Kuo, 1998), the generalized gamma distribution (Bak et al., 2002; Corral, 2004; Shcherbakov et al., 2005), the log normal distribution (Matthews et al., 2002), the Poissonian distribution (Dionysiou and Papadopoulos, 1992), the negative binomial distribution (Madhava Rao and Kaila, 1986), the Gaussian distribution (Papazachos et al., 1987) and the Bayesian distribution (Papadopoulos, 1987) were used for candidates of the distribution functions of interoccurrence and recurrence times. However, the most appropriate distribution function of the interoccurrence and recurrence time remains under debate and open. Utsu (1984), for instance, applied four probability models to analyze interoccurrence times for Japanese earthquakes and discussed their significances. Recently, in the stationary regime, a unified scaling law of the interoccurrence time statistics was proposed by Corral (2004). Abe and Suzuki (2005) on the other hand showed that the cumulative distribution of interoccurrence times is very well fitted by the q-exponential distribution ($q>1$) corresponding to the power law. Two underlying assumptions should be noticed in those papers: (a) Earthquakes can be considered as a point process in space and time; (b) There is no distinction between foreshocks, mainshocks, and aftershocks.

Except for real earthquake data used in abovementioned papers, due to the limitation of real earthquake data, the time-interval statistics have also been studied by means of numerical simulations of earthquake models (e.g. Rundle et al., 2000; Abaimov et al., 2007; Hasumi, 2007; Hasumi et al., 2009a). Both the conceptual spring-block models (Abaimov et al., 2007; Hasumi et al., 2009a) and the sophisticated Virtual California model (Yakovlev et al., 2006) show the Weibull distribution of the recurrence times. Hasumi (2007) reported that the cumulative distribution of interoccurrence times in the two-dimensional (2D) spring-block model can be described as the Zipf–Mandelbrot type power law which has been early observed by Abe and Suzuki (2005).

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Yet another new statistical feature on the interoccurrence times, the Weibull–log Weibull transition, was very recently proposed by analyzing the Japan Meteorological Agency (JMA) catalogue (Hasumi et al., 2009b). Hasumi et al. (2009b) found that the probability distribution of interoccurrence times can be very well fitted by the superposition of the log Weibull distribution and the Weibull distribution. The results in Hasumi et al. (2009b) demonstrate that the interoccurrence time statistics probably contain both the Weibull and log Weibull statistics and, as the threshold of magnitude m_c increases, the predominant distribution could change from the log Weibull distribution to the pure Weibull distribution. The distribution of large earthquakes obeys the Weibull distribution with an exponent less than unity indicating that the process of large earthquakes is not a Poissonian process. Importantly, those hybrid distributions and the Weibull–log Weibull transition can be also found in synthetic catalogues produced by the 2D spring-block model (Hasumi et al., 2009a). The applicability to other tectonic regions of the Weibull–log Weibull transition however remains unsolved. Whether or not is the Weibull–log Weibull transition universal?

In this study we investigate the interoccurrence time statistics by analyzing the Southern California and Taiwan earthquake catalogues. Together with the previous results from the JMA and synthetic catalogues shown in Hasumi et al. (2009a,b), a universal Weibull–log Weibull transition can be obtained in all of these catalogues. We also suggest that a crossover magnitude m_c^{**} between the superposition regime and the pure Weibull regime is proportional to the plate velocity and, at the end of this paper, we elucidate its implication in the geophysical sense.

2. Data and methodology

For studying the interoccurrence time statistics we analyzed three natural earthquake catalogues of the Japan Metrological Agency (JMA), the Southern California Earthquake Data Center (SCEDC) and the Taiwan Central Weather Bureau (TCWB), as well as one synthetic catalogue generated from the 2D spring-block model. Information on each catalogue are listed in Table 1, where $m_{\min}$ corresponds to the minimum magnitude in the catalogue and m_c^0 is the magnitude of completeness, that is the lowest magnitude at which the Gutenberg–Richter law holds. We basically consider events with magnitude greater than and equal to m_c^0 because events smaller than m_c^0 are supposedly incomplete for recording.

The synthetic catalogue is produced by the 2D spring-block model with the velocity-weakening friction law (Carlson et al., 1991). The 2D spring-block model is characterized by five parameters: the stiffness l_x^2 and l_y^2, the decrement of the friction force α, the plate velocity ν, and the difference between the maximum friction force and dynamical friction force σ. We have set those parameters as $l_x^2=1$, $l_y^2=3$, $\alpha=3.5$, $\nu=0.01$ and $\sigma=0.01$, which make the model reproduce several realistic aspects of events in the Gutenberg–Richter relation with a b-value of 1 (Kumagai et al., 1999; Hasumi, 2007), the constant stress drop (Kumagai et al., 1999), the interoccurrence time statistics (Hasumi, 2007) and the hypocenter interval statistics (Hasumi, 2009). For many details on simulation of the 2D spring-block model, we refer the readers to the papers of Hasumi (2007, 2009). Event magnitude m in the model is defined as $m = m_0 + \log_{10}\left(\sum_{i,j}^{n} \delta u_{i,j}\right) / 1.5$, where $\delta u_{i,j}$ and n are the total slip at the cell (i, j) and the total number of slipping blocks, respectively. m_0 is set at 0.7 for shifting m to a positive value. The occurrence time of an event is the simulating time step when the beginning block slips during an event. The nth interoccurrence time τ_n is defined as $t_{n+1}-t_n$, where t_n and t_{n+1} are occurrence times of the nth and $n+1$th events, respectively.

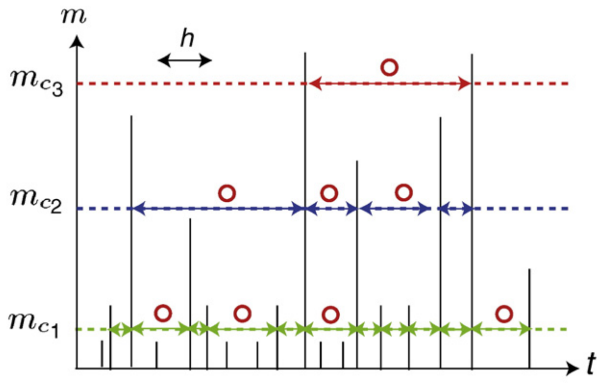

Fig. 1. The schematic diagram for our analysis. m_{c1}, m_{c2}, and m_{c3} are different thresholds of magnitude. We consider the interoccurrence time distribution in the time domain $\tau>h$, corresponding to °.

We show the schematic illustration of our analyzing procedure in Fig. 1. The procedure is the same as that used in the previous studies by Hasumi et al. (2009a,b) and is briefly explained as following: (a) We divided the studied region into the spatial windows with the size of $L\times L$; (b) For each spatial domain, events larger than a certain threshold of magnitude m_c were considered; (c) We calculated interoccurrence times and then performed the distribution fitting over the interoccurrence data τ_i larger than h days. Note that the procedure (a) is used for the JMA catalogue only, because its coverage is much bigger than those of the SCEDC and TCWB catalogues. Also, we focus on the interoccurrence time statistics for long time domain for eliminating the aftershock effect. Although there exist several de-clustering algorithms (e.g. Davis and Frohlich, 1991), the best way for removing aftershocks from the catalogue remains under debate still. We have therefore introduced the temporal parameter h in the procedure (c) for eliminating the immediately time-correlated events. Same strategy has been utilized in Corral (2004) and in Enescu et al. (2008), except we set a larger h value of 0.5 for real catalogues. As for the synthetic catalogue, the 2D spring-block model without the viscous factor does not produce aftershocks. The procedure (c) is therefore skipped, corresponding to $h=0$.

An important goal in this study is to detect the change in the probability distribution of interoccurrence time $P(\tau)$ by varying the magnitude threshold m_c. Here, same as the previous works by Hasumi et al. (2009a,b), we consider five candidate functions for $P(\tau)$, namely, the Weibull distribution $P_{\rm w}$, the log Weibull distribution $P_{\rm lw}$, the power law distribution $P_{\rm pow}$, the gamma distribution $P_{\rm gam}$ and the log normal distribution $P_{\rm ln}$. The probability density functions of these distributions are

$$P_{\rm w}(\tau) = \left(\frac{\tau}{\beta_1}\right)^{\alpha_1-1}\frac{\alpha_1}{\beta_1}\exp\left[-\left(\frac{\tau}{\beta_1}\right)^{\alpha_1}\right], \qquad (1)$$

Table 1
Information on the used earthquake catalogues.

Catalogue name	Coverage	Term	Number of earthquakes	$m_{\min}$	m_c^0	$m_{\max}$
JMA	25°–50°N and 125°–150°E	01/01/2001–31/10/2007	130,244	2.0	2.0	8.0
SCEDC	32°–37°N and 114°–122°W	01/01/2001–31/12/2007	10,838	0.0	1.4	5.7
TCWB	21°–26°N and 119°–123°E	01/01/2001–31/12/2007	148,155	0.0	1.9	7.1
Synthetic	50×50 (System size)	–	297,040	0.0	0.3	2.8

Table 2
Results of rms value, *DKS*, and *Q*, and parameters for different distribution functions in the case of $m_c = 3.5$ for California earthquakes.

Distribution *X*	α_i	β_i	rms [$\times 10^{-3}$]	*DKS*	*Q*
P_w ($i=1$)	0.95 ± 0.01	9.46 ± 0.08	7.6	0.026	0.999
P_{lw} ($i=2$)	2.84 ± 0.10	17.6 ± 0.50	27	0.112	0.125
P_{pow} ($i=3$)	1.57 ± 0.05	0.87 ± 0.03	11	0.229	1.89×10^{-5}
P_{gam} ($i=4$)	0.99 ± 0.005	9.53 ± 0.09	9.8	0.030	0.999
P_{ln} ($i=5$)	1.81 ± 0.03	1.05 ± 0.03	24	0.084	0.403

Table 4
Results of rms value, *DKS*, and *Q*, and parameters for different distribution functions in the case of $m_c = 4.5$ for Taiwanese earthquakes.

Distribution *X*	α_i	β_i	rms [$\times 10^{-3}$]	*DKS*	*Q*
P_w ($i=1$)	0.92 ± 0.01	5.44 ± 0.04	5.8	0.019	1
P_{lw} ($i=2$)	2.28 ± 0.12	9.93 ± 0.21	31	0.129	0.13
P_{pow} ($i=3$)	1.67 ± 0.07	0.66 ± 0.12	11	0.200	0.003
P_{gam} ($i=4$)	0.99 ± 0.005	5.52 ± 0.07	10	0.040	0.999
P_{ln} ($i=5$)	1.25 ± 0.04	1.07 ± 0.04	22	0.079	0.690

$$P_{lw}(\tau) = \frac{(\log(\tau/k))^{\alpha_2 - 1}}{(\log\beta_2)^{\alpha_2}} \frac{\alpha_2}{\tau} \exp\left[-\left(\frac{\log(\tau/k)}{\log\beta_2}\right)^{\alpha_2}\right], \quad (2)$$

$$P_{pow}(\tau) = \frac{1}{(1+\beta_3\tau)^{\alpha_3}}, \quad (3)$$

$$P_{gam}(\tau) = \tau^{\alpha_4 - 1} \frac{\exp(-\tau/\beta_4)}{\Gamma(\alpha_4)\beta_4^{\alpha_4}}, \quad (4)$$

$$P_{ln}(\tau) = \frac{1}{\tau\beta_5\sqrt{2\pi}} \exp\left[-\frac{(\ln(\tau)-\alpha_5)^2}{2\beta_5^2}\right], \quad (5)$$

where α, β and k are constants characterizing these distributions. $\Gamma(x)$ is the gamma function. i stands for an index number; $i = 1, 2, 3, 4$ and 5 corresponding to the Weibull distribution, the log Weibull distribution, the power law distribution, the gamma distribution and the log normal distribution, respectively. k is fixed at 0.5 throughout this work. Note that the log Weibull distribution is derived by the logarithmic modification of the cumulative distribution of the Weibull distribution and reduces to the power law distribution as α_2 is unity.

For determining which distribution can fit better the interoccurrence data, we used the root mean square (rms) and Kolomogorov–Smirnov (KS) tests as the measure of goodness-of-fit. The definition of the rms value is

$$rms = \sqrt{\frac{\sum_{i=1}^{n} (x_i - x_i')^2}{n' - k'}}, \quad (6)$$

where x_i are actual data and x_i' are estimated data obtained from $P(\tau)$. n' and k' indicate the numbers of the data points and of the fitting parameters, respectively. In this study, the rms value is calculated using the cumulative distribution for decreasing the fluctuation of the data. The most appropriate distribution is by definition with the smallest rms value. Also, in order to use the KS test, we define the maximum deviation of static *DKS* as

$$DKS = \max_i |y_i - y_i'|, \quad (7)$$

where y_i and y_i' mean the actual data of the cumulative distribution and the data estimated from the fitting distribution, respectively. Then, the significance level of probability of the goodness-of-fit, Q, is defined as

$$Q = 2\sum_{i=1}^{\infty} (-1)^{i-1} e^{-2i^2\lambda^2}, \quad (8)$$

where

$$\lambda = DKS\left(\sqrt{n'} + 0.12 + \frac{0.11}{\sqrt{n'}}\right). \quad (9)$$

It is recognized that the preferred distribution show the smallest value of *DKS* and the largest value of *Q* (Press et al., 1995).

3. Results

We here investigate the interoccurrence time statistics by analyzing the SCEDC and the TCWB data. The results are listed in Tables 2 and 3 for SCEDC and Tables 4 and 5 for TCWB. As for the results from the JMA and synthetic data, we refer the readers to the previous works by Hasumi et al. (2009a,b). Hasumi et al. (2009a,b) have demonstrated that, in the JMA and synthetic data, the

Table 3
Interoccurrence time statistics by analyzing the SCEDC data. ± means the 95% confidence level of fit.

m_c	Distribution *X*	Weibull distribution		Distribution X		*p*	rms [$\times 10^{-3}$]	Kolmogorov–Smirnov test	
		α_1	β_1	α_i	β_i			*DKS*	*Q*
4.0	P_{lw} ($i=2$)	0.98 ± 0.02	26.7 ± 0.52	–	–	1	23	0.076	0.421
	P_{pow} ($i=3$)	0.98 ± 0.02	26.7 ± 0.52	–	–	1	23	0.076	0.421
	P_{gam} ($i=4$)	0.98 ± 0.02	26.7 ± 0.52	0.99 ± 0.01	26.7 ± 0.50	0.34 ± 2.58	23	0.076	0.421
	P_{ln} ($i=5$)	0.98 ± 0.02	26.7 ± 0.52		–	1	23	0.076	0.421
3.5	P_{lw} ($i=2$)	0.95 ± 0.01	9.46 ± 0.08	–	–	1	7.6	0.026	0.999
	P_{pow} ($i=3$)	0.95 ± 0.01	9.46 ± 0.08	–	–	1	7.6	0.026	0.999
	P_{gam} ($i=4$)	0.95 ± 0.01	9.46 ± 0.08	–	–	1	7.6	0.026	0.999
	P_{ln} ($i=5$)	0.95 ± 0.01	9.46 ± 0.08		–	1	7.6	0.026	0.999
3.0	P_{lw} ($i=2$)	1.00 ± 0.02	3.41 ± 0.10	1.52 ± 0.14	3.75 ± 0.38	0.77 ± 0.02	4.1	0.014	1
	P_{pow} ($i=3$)	0.97 ± 0.02	3.08 ± 0.04	1.82 ± 0.06	0.59 ± 0.06	0.91 ± 0.02	6.8	0.046	0.992
	P_{gam} ($i=4$)	0.97 ± 0.02	3.08 ± 0.04	0.99 ± 0.004	3.07 ± 0.02	0.09 ± 0.60	9.1	0.056	0.941
	P_{ln} ($i=5$)	0.97 ± 0.02	3.08 ± 0.04	0.71 ± 0.02	1.01 ± 0.04	0.57 ± 0.06	5.4	0.028	0.999
2.5	P_{lw} ($i=2$)	1.44 ± 0.02	1.45 ± 0.02	1.22 ± 0.04	2.16 ± 0.04	$0.58. \pm 0.01$	2.3	0.011	1
	P_{pow} ($i=3$)	1.44 ± 0.06	1.35 ± 0.02	2.36 ± 0.08	0.52 ± 0.02	$0.72. \pm 0.02$	7.2	0.029	0.999
	P_{gam} ($i=4$)	1.44 ± 0.06	1.35 ± 0.02	–	–	1	17	0.083	0.554
	P_{ln} ($i=5$)	–	–	0.02 ± 0.06	0.70 ± 0.02	0	6.1	0.043	0.996
2.0	P_{lw} ($i=2$)	1.83 ± 0.04	0.81 ± 0.01	1.13 ± 0.02	1.47 ± 0.01	0.45 ± 0.01	1.7	0.007	1
	P_{pow} ($i=3$)	2.36 ± 0.16	0.81 ± 0.01	3.40 ± 0.10	0.48 ± 0.008	0.44 ± 0.04	7.1	0.027	1
	P_{gam} ($i=4$)	2.36 ± 0.16	0.81 ± 0.01	1.11 ± 0.06	0.72 ± 0.06	0.97 ± 0.10	25	0.121	0.237
	P_{ln} ($i=5$)	–	–	-0.38 ± 0.01	0.42 ± 0.04	0	15	0.078	0.078

Table 5
Interoccurrence time statistics by analyzing the TCWB data. ± means the 95% confidence level of fit.

m_c	Distribution X	Weibull distribution		Distribution X		p	rms [$\times 10^{-3}$]	Kolmogorov–Smirnov test	
		α_1	β_1	α_i	β_i			*DKS*	*Q*
5.0	P_{lw} $(i=2)$	0.86 ± 0.02	15.3 ± 0.22	–	–	1	12	0.039	0.997
	P_{pow} $(i=3)$	0.86 ± 0.02	15.3 ± 0.22	–	–	1	12	0.039	0.997
	P_{gam} $(i=4)$	0.86 ± 0.02	15.3 ± 0.22	–	–	1	12	0.039	0.997
	P_{ln} $(i=5)$	0.86 ± 0.02	15.3 ± 0.22		–	1	12	0.039	0.997
4.5	P_{lw} $(i=2)$	0.92 ± 0.01	5.44 ± 0.04	–	–	1	5.8	0.019	1
	P_{pow} $(i=3)$	0.92 ± 0.01	5.44 ± 0.04	–	–	1	5.8	0.019	1
	P_{gam} $(i=4)$	0.92 ± 0.01	5.44 ± 0.04	–	–	1	5.8	0.019	1
	P_{ln} $(i=5)$	0.92 ± 0.01	5.44 ± 0.04		–	1	5.8	0.019	1
4.0	P_{lw} $(i=2)$	1.00 ± 0.04	2.20 ± 0.10	1.84 ± 0.14	4.14 ± 0.34	0.77 ± 0.02	3.5	0.0100	1
	P_{pow} $(i=3)$	1.09 ± 0.02	2.25 ± 0.04	1.94 ± 0.08	0.53 ± 0.06	0.88 ± 0.02	7.4	0.029	0.999
	P_{gam} $(i=4)$	1.09 ± 0.02	2.25 ± 0.04	2.04 ± 0.02	0.98 ± 0.04	0.83 ± 0.06	9.4	0.028	0.999
	P_{ln} $(i=5)$	1.09 ± 0.02	2.25 ± 0.04	0.44 ± 0.01	0.91 ± 0.01	0.40 ± 0.06	3.4	0.0102	1
3.5	P_{lw} $(i=2)$	1.44 ± 0.06	1.28 ± 0.02	1.24 ± 0.10	2.08 ± 0.12	0.64 ± 0.04	5.0	0.010	1
	P_{pow} $(i=3)$	1.50 ± 0.06	1.24 ± 0.02	2.42 ± 0.10	0.50 ± 0.02	$0.73. \pm 0.04$	8.4	0.027	1
	P_{gam} $(i=4)$	–	–	2.01 ± 0.01	0.56 ± 0.02	0	14	0.085	0.778
	P_{ln} $(i=5)$	–	–	-0.05 ± 0.006	0.66 ± 0.008	0	6.1	0.028	1
3.0	P_{lw} $(i=2)$	2.08 ± 0.14	0.75 ± 0.02	1.24 ± 0.10	1.49 ± 0.02	0.47 ± 0.04	3.4	0.012	1
	P_{pow} $(i=3)$	2.63 ± 0.20	0.78 ± 0.01	3.56 ± 0.14	0.48 ± 0.008	0.53 ± 0.04	7.5	0.026	1
	P_{gam} $(i=4)$	2.63 ± 0.20	0.78 ± 0.01	1.92 ± 0.10	0.37 ± 0.04	0.94 ± 0.12	24	0.101	0.50
	P_{ln} $(i=5)$	–	–	-0.40 ± 0.008	0.38 ± 0.01	0	12	0.051	0.997
2.5	P_{lw} $(i=2)$	3.35 ± 1.08	0.59 ± 0.06	1.01 ± 0.18	1.18 ± 0.04	0.40 ± 0.12	14	0.032	1
	P_{pow} $(i=3)$	5.09 ± 0.72	0.61 ± 0.01	6.21 ± 0.22	0.48 ± 0.004	0.39 ± 0.04	14	0.063	0.999
	P_{gam} $(i=4)$	5.09 ± 0.72	0.61 ± 0.01	1.67 ± 4.84	0.68 ± 0.73	0.96 ± 0.14	43	0.131	0.715
	P_{ln} $(i=5)$	–	–	-0.57 ± 0.01	0.19 ± 0.02	0	32	0.086	0.983

interoccurrence time distribution $P(\tau)$ for the large magnitude threshold m_c can be best fitted by the Weibull distribution among all five distributions in Eqs. (1)–(5), which is supported by the rms test and KS test for the synthetic data (Hasumi et al., 2009a) and by the rms test, KS test, and Anderson–Darling (AD) test for the JMA data (Hasumi et al., 2009b). As shown in Tables 2 and 4, for the SCEDC and TCWB data, the interoccurrence time distribution $P(\tau)$ with large m_c, again, can be very well fitted by the Weibull distribution among the candidates, because the Weibull distribution possesses the smallest rms and *DKS* values, and the largest *Q* value. This result is basically consistent with the results in Abaimov et al. (2007, 2008). They have revealed that the interoccurrence time statistics of large events can be well described by the Weibull distribution by analyzing the Parkfield and Writewood earthquake data (Abaimov et al., 2008), and the synthetic catalogue created by the one-dimensional (1D) spring-block model (Abaimov et al., 2007).

However, as m_c is gradually decreased, the goodness-of-fit of the Weibull distribution becomes worse (see Tables 3 and 5). We thus propose a conjecture that the interoccurrence time distribution could be better described by the superposition of the Weibull distribution and another distribution *X*, i.e.

$$P(\tau) = p \times \textit{Weibull distribution} + (1-p) \times \textit{distribution } X, \qquad (10)$$

where *p* is the percentage of the Weibull distribution in $P(\tau)$ and the candidates of the distribution *X* are abovementioned the log Weibull distribution, the power law distribution, the gamma distribution and the log normal distribution. We furthermore find that in all investigated m_c domains, due to the lowest rms and *DKS* values, and the largest *Q* value (Tables 3 and 5), the log Weibull distribution is the most appropriate for the distribution *X*. We hence conclude that $P(\tau)$ can be very well described by using the superposition of the Weibull and log Weibull distributions:

$$P(\tau) = p \times \textit{Weibull distribution} + (1-p) \times \log \textit{Weibull distribution}. \qquad (11)$$

Obviously $P(\tau)$ is $P_{lw}(\tau)$ when $p=0$ and is $P_w(\tau)$ when $p=1$. Results shown in Tables 3 and 5 thus bring us a view that the interoccurrence time statistics contain both the Weibull statistics and the log Weibull statistics and, as m_c is gradually increased, the dominant distribution of $P(\tau)$ then changes from the log Weibull distribution to the Weibull distribution. That means $P(\tau)$ must exhibit the Weibull–log Weibull transition.

As an example, together with the previous results of the JMA and synthetic catalogues (Hasumi et al., 2009a,b), we show the cumulative distributions of interoccurrence times for the log Weibull regime, the superposition regime and the Weibull regime in Fig. 2(a), (b) and (c), respectively. To make clear the log Weibull and Weibull regimes, we demonstrate the log Weibull plot for the log Weibull fit and the Weibull plot for the Weibull fit in Fig. 2(d) and (e), respectively. Note that those results of the JMA data shown in Fig. 2 are obtained from a divided region spanning 35°–40°N and 140°–145°E. In addition, the pure log Weibull regime can only be observed in the synthetic data (Fig. 2a) because the real catalogues undoubtedly lack for events much smaller than the magnitude of completeness m_c^0. Although we cannot detect the pure log Weibull regime for the real catalogues, the superposition regime as shown in Fig. 2(b) nevertheless suggests that the transition from the log Weibull distribution to the Weibull distribution in the interoccurrence times could appear universally in different tectonic settings.

The changes in the fitting parameters of the Weibull and log Weibull distributions, as a function of m_c/m_{max}, are shown in Figs. 3 and 4. We have introduced the scaled magnitude, defined as m_c/m_{max}, for unifying the data presentation of individual catalogues. We demonstrate the relation between the Weibull parameters (α_1, β_1) as a function of m_c/m_{max} in Fig. 3. As shown in Fig. 3(a), α_1 gradually decreases as m_c/m_{max} increases for the case of JMA, SCEDC and TCWB. Additionally, for large m_c/m_{max}, α_1 obtained from the natural data is less than unity, indicating that the occurrence of natural earthquakes is not a Poissonian process, whereas α_1 for the synthetic data gradually increases and is greater than unity. However, it should be also noted that α_1 derived from the synthetic data could be less than 1 by tuning the parameters of the spring-block model (Hasumi et al., 2009a). On the other hand, as m_c/m_{max} increases, β_1 for the synthetic data (Fig. 3b) increases double exponentially while β_1 for natural data (Fig. 3c) increases exponentially. We also show in Fig. 4 the log Weibull parameters (α_2, β_2) as a function of m_c/m_{max}. For all catalogues, as m_c/m_{max} increases, α_2 (Fig. 4a) and β_2 (Fig. 4b) increase linearly and exponentially, respectively.

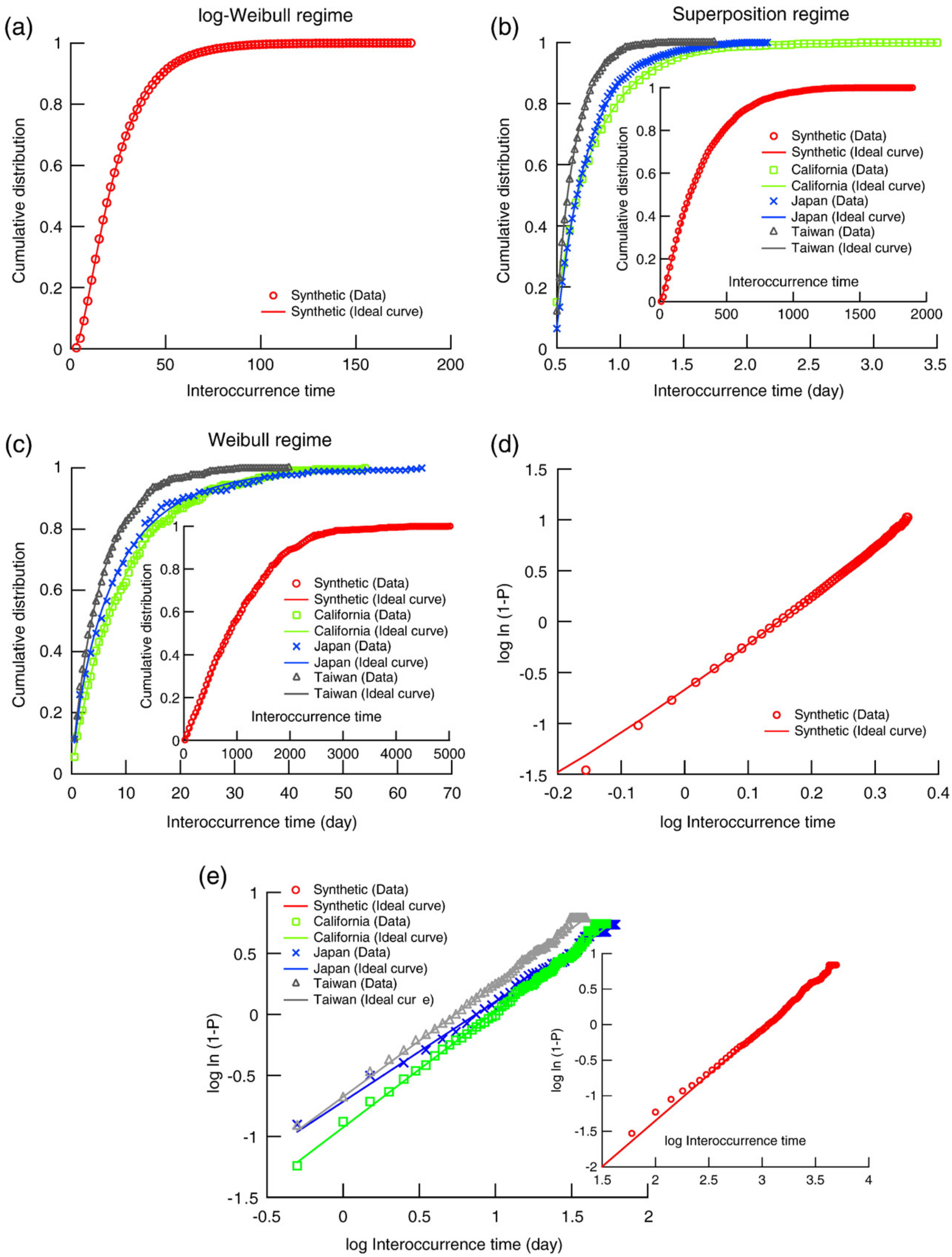

Fig. 2. The cumulative distribution of interoccurrence times for different catalogues. (a), (b) and (c) correspond to the log Weibull regime, the superposition regime and the Weibull regime, respectively. For (b) and (c), the result of the synthetic data is displayed in the inset figures. (d) and (e) are the log Weibull and Weibull plots, respectively.

Most importantly, as clearly shown in Fig. 5, the percentage p of the Weibull distribution in $P(\tau)$ gradually increases as m_c/m_{max} increases, which demonstrates that as the threshold of magnitude is varied the probability distribution for $P(\tau)$ changes and the dominant distribution is transferred from the log Weibull distribution to the Weibull distribution. We thus define two crossover (transition)

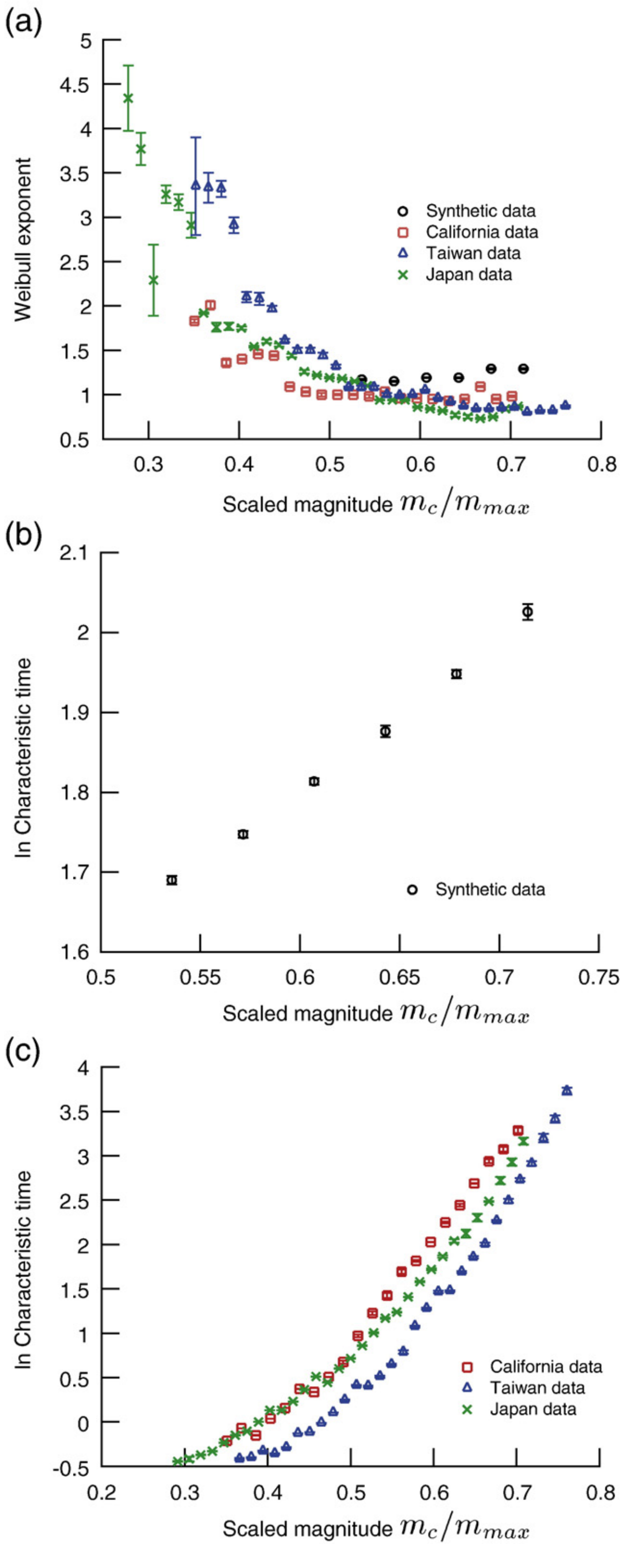

Fig. 3. Change of the Weibull parameters as the function of m_c/m_{max}. The result of α_1, β_1 for the synthetic, and for natural data are shown in (a), (b) and (c), respectively.

magnitudes m_c^*/m_{max} and m_c^{**}/m_{max} representing the scaled magnitudes of the distribution changed from the pure log Weibull regime to the superposition regime and from the superposition regime to the pure Weibull regime, respectively (Fig. 5). As shown in Fig. 5, the transition from the log Weibull regime to the Weibull regime for the synthetic data (circles) appears clearly, so that both the first (m_c^*/m_{max}) and the second (m_c^{**}/m_{max}) crossover magnitudes can be well

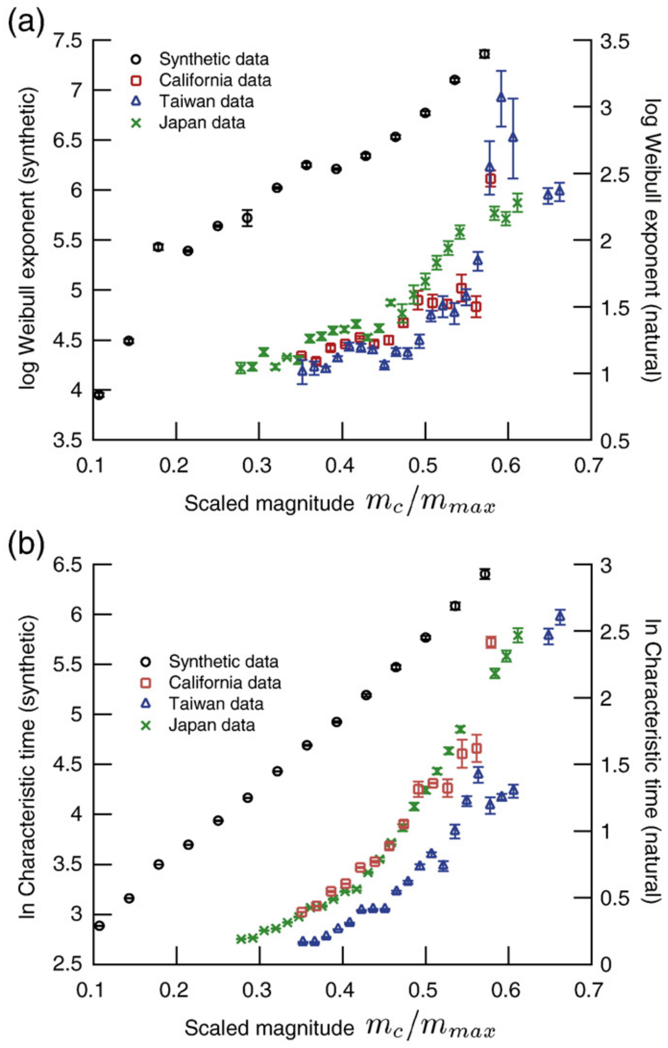

Fig. 4. Change of the log Weibull parameters as the function of m_c/m_{max}. The result of α_2 and β_2 are presented in (a) and (b), respectively.

estimated. As for the cases of natural catalogues, due to incomplete events smaller than the magnitude of completeness, the pure log Weibull regime cannot be observed (Fig. 5) and we can thus

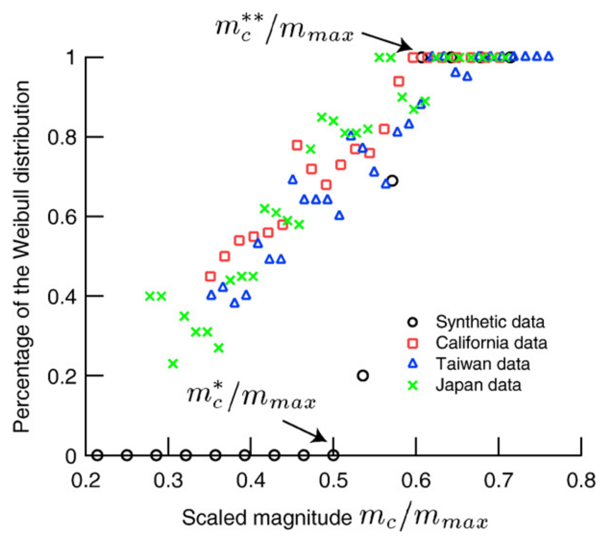

Fig. 5. Change of the percentage of the Weibull distribution as the function of m_c/m_{max}.

determine the second crossover point m_c^{**}/m_{max} only. It looks like that the values of m_c^{**}/m_{max} are coincidently around 0.60 for all of four catalogues, indicating its region-independence.

4. Discussion

To study the feature of the Weibull–log Weibull transition, we summarize our results obtained from 17 different regions (14 regions in Japan, 2 regions of Southern California and Taiwan, and a virtual region of the 2D spring-block model) in Fig. 6. We interestingly find that m_c^{**} is proportional to the maximum magnitude (Fig. 6a). We then obtain a region-independent constant for the scaled crossover magnitude, namely, $m_c^{**}/m_{max} = 0.54 \pm 0.06$ (Fig. 6b). Note that, in Fig. 6(b), there are three outliers with the values of m_c^{**}/m_{max} less than 0.5, corresponding to those regions of (30°–35°N and 125°–130°E), (45°–50°N and 140°–145°E) and (40°–45°N and 140°–145°E) in Japan. The numbers of earthquakes used in the first two regions are in the order of 10^2 and are much smaller than the earthquake numbers used in other regions. Therefore, the small values of m_c^{**}/m_{max} are probably caused by insufficient statistical samplings. As for the region of (40°–45°N and 140°–145°E), m_{max} is 8.0 which is the largest magnitude throughout the JMA catalogue we analyzed. Therefore m_c^{**}/m_{max} for this region tends to be downward biased.

Further investigation shows that, although the scaled crossover magnitudes m_c^{**}/m_{max} is region-independent, the crossover magnitude m_c^{**} from the superposition regime to the pure Weibull regime probably depends on the tectonic region (Table 6). Comparing the plate velocity with *averaged* m_c^{**} sheds light on the geophysical implication of the region-dependent m_c^{**}. As shown in Table 6, m_c^{**} is on the average proportional to the plate velocity. That means the maximum magnitude m_{max} for a tectonic region is more or less proportional to the plate velocity since $m_c^{**}/0.54 = m_{max}$. Such an interesting consequence is reminiscent of the early study by Ruff and Kanamori (1980). Ruff and Kanamori (1980) showed a relation indicating that the magnitude of characteristic earthquake occurred in the subduction-zone is directly proportional to the convergence rate. The relation $m_c^{**}/0.54 = m_{max}$ we find in the present study can thus be explained on the basis of their early observation about the velocity-dependence of the characteristic earthquake magnitude.

The physical interpretation of the Weibull–log Weibull transition remains open. However, it might suggest that the occurrence mechanism of earthquake could probably depend on its magnitude then, inevitably, the probability distribution of the interoccurrence time statistics changes as the threshold of magnitude m_c is varied. It is well known that the Weibull distribution for life-time of materials can be derived in the framework of damage mechanics (Weibull, 1951; Ghosh, 1999; Turcotte et al., 2003; Wong et al., 2006; Abaimov et al., 2007). Our present results thus suggest that larger earthquakes might be caused by the damage mechanism driven by the plate motion, whereas the effect of the plate-driven damaging process might become minor for smaller earthquakes. Hence, the transition of the Weibull regime to the log Weibull regime could be interpreted from the geophysical sense as the decrement of the plate-driven damaging mechanics.

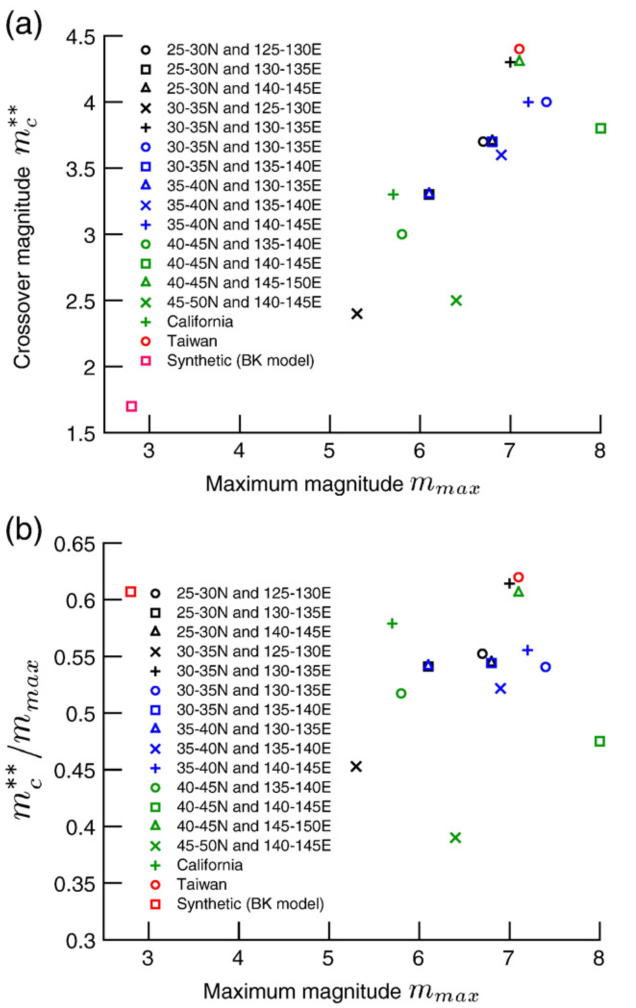

Fig. 6. Relation between m_c^{**} and m_{max}. In (a), m_c^{**} is proportional to the m_{max}. In (b), we obtain a region-independent constant for the scaled crossover magnitude $m_c^{**}/m_{max} = 0.54 \pm 0.06$.

5. Concluding remarks

We have investigated the interoccurrence time statistics of natural and synthetic earthquakes by analyzing the JMA, SCEDC, TCWB and synthetic catalogues. We emphasize the interoccurrence time statistics contain both the Weibull and log Weibull distributions. And, in this paper, we demonstrate the universal Weibull–log Weibull transition in the interoccurrence time distributions for different tectonic settings. Our present work represents the first step to fully understand the interoccurrence time statistics and the Weibull–log Weibull transition for real earthquakes. In this study, we also propose the region-independent scaling relation, namely, $m_c^{**}/m_{max} = 0.54 \pm 0.06$. We find the crossover magnitude m_c^{**} is proportional to the plate velocity, which is consistent with an earlier observation about the velocity-dependence of the characteristic earthquake magnitude (Ruff and Kanamori, 1980). Although the origins of both the log Weibull distribution and the Weibull–log Weibull transition remain open, we suggest the change in the distribution from the log Weibull distribution to the Weibull distribution can be considered as the enhancement in the plate-driven damaging mechanics.

Table 6
List of the crossover magnitude and the plate velocity (Fowler, 1990; Seno et al., 1993). The notation of PH, EU, PA, and NA represent PHilippine Sea plate, EUrasian plate, PAcific plate, and North American plate, respectively.

Region	Relative plate motion	Velocity [mm/yr]	m_c^{**}
Taiwan	PH–EU	71	4.40
East Japan	PA–PH	49	3.80[a]
West Japan	PH–EU	47	3.80[b]
California	PA–NA	47	3.40

[a] We take an average using three regions; 25°–30°N and 140°–145°E ($m_c^{**} = 3.7$), 30°–35°N and 140°–145°E ($m_c^{**} = 3.7$), and 35°–40°N and 140°–145°E ($m_c^{**} = 4.0$).

[b] We take an average using five regions; 25°–30°N and 125°–130°E ($m_c^{**} = 3.7$), 25°–30°N and 130°–135°E ($m_c^{**} = 3.3$), 30°–35°N and 130°–135°E ($m_c^{**} = 4.3$), 30°–35°N and 135°–140°E ($m_c^{**} = 4.1$), and 35°–40°N and 135°–140°E ($m_c^{**} = 3.6$).

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