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Quasi-periodicity of large avalanches in the long-range connective sandpile models and its implication in natural earthquakes

YA-TING LEE[1(a)], CHIEN-CHIH CHEN[1], CHAI-YU LIN[2] and YA-TING CHUANG[1]

[1] *Department of Earth Science and Graduate Institute of Geophysics, National Central University Jhongli, Taiwan 320, ROC*
[2] *Dept Physics, National Chung-Cheng University - Ming-Hsiung, Chia-Yi, Taiwan 621, ROC*

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Abstract – In this study, we investigated the quasi-periodicity of large avalanches using a new modification of sandpile models, *i.e.*, the long-range connective sandpile (LRCS) model. The LRCS model considers the random distant connection between two separated, instead of neighboring, cells and shows interesting precursory behavior before large avalanches. We explore the statistics of recurrence intervals between large events and find a strong dependence on the size L of the sandpile. In the LRCS model, the average recurrence interval W of large avalanches follows the relationship $W \propto L^{2.07}$. Having the recurrence intervals of many earthquake fault systems around the world, we propose an empirical rule between those intervals and the corresponding earthquakes' magnitudes and provide evidence of the quasi-periodic behavior of natural earthquakes.

Introduction. – For natural earthquake fault systems, the quasi-periodicity remains an open issue and has great societal importance. Large earthquakes are not periodic; however, they could be quasi-periodic. A typical example showing quasi-periodicity is in the Parkfield, California area. The San Andreas Fault is the primary boundary between the Pacific and the North American plates, and the displacements associated with it are distributed over many fault segments. Studies on the San Andreas Fault have led to the conception of the *characteristic earthquake* model. The earthquake sequence occurred in the Parkfield segment of the San Andreas Fault in 1857, 1881, 1901, 1922, 1934, and 1966. This is an excellent example of a moderate characteristic earthquake with an average recurring interval of 22 years [1]. These earthquakes repeat with similar features, including faulting mechanisms, epicenter locations, magnitudes, seismic moments, rupture areas, and southeast rupture propagations. The Parkfield earthquake sequence has given seismologists an opportunity to test the applicability of recurrence models in regions characterized by recurring moderate-sized earthquake. Bakun and Lindh [1] thus predicted in 1985 that the next moderate Parkfield earthquake after 1966 would occur around 1988 and not later than 1993. The earthquake, however, did not occur within the predicted timescale. Surprisingly, in September 2004, an earthquake occurred in Parkfield with comparable magnitude. There was much debate about whether the event that occurred in 2004 was the expected characteristic earthquake and, if so, why it happened 11 years later. We present in this study an answer to this important issue about earthquake recurrences.

Sandpile dynamics and self-organized criticality (SOC) are exhibited in many natural and social phenomena, including earthquakes, forest fires, rainfalls, landscapes, drainage networks, stock prices, and traffic jams. Since Bak *et al.* [2,3] introduced the original nearest-neighboring sandpile model, various numerical and analytical studies of modified sandpile models have been the subject of much research (*e.g.*, [4–12]). Among these approaches, the *annealed* random-neighbor sandpile models in which an avalanche can propagate within the system were first proposed by Christensen and Olami [5] and then extensively

(a) E-mail: shine2530@gmail.com

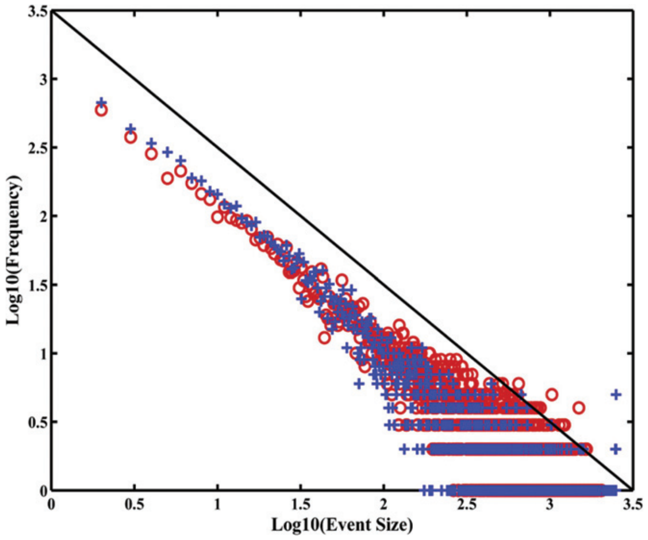

Fig. 1: (Colour on-line) The power-law frequency-size distributions of avalanches for the original Bak-Tang-Wiesenfeld–type sandpile (red circles) and the modified long-range connective sandpile (blue crosses) models. The power-law distribution with a slope of 1 (diagonal line) can fit the data from the original sandpile model and the modified long-range connective sandpile model with variable P_c [12].

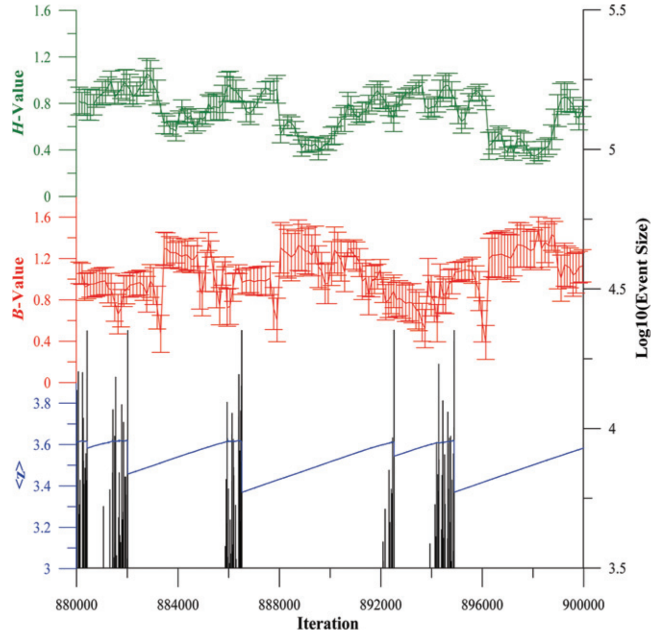

Fig. 2: (Colour on-line) An LRCS simulation for a square lattice of 150 by 150 cells. The blue line represents the dynamic variable $\langle Z \rangle(t)$ for the average topographic height of the board in the LRCS model. The green and red lines are the Hurst exponent H for avalanche sizes and the power-law exponent B for the frequency-size distributions, respectively. The error bars show the 95% confidence intervals. The time occurrences for avalanches with $s > 10^{3.5}$ (black bars) are also shown [15].

studied on a long-range connected (small-world) network by de Arcangelis and Herrmann [8], Lahtinen *et al.* [10], and Chen *et al.* [11,12].

Long-range connective sandpile model. – We previously proposed a *long-range connective sandpile* (LRCS) model by introducing randomly remote connections between two separated, instead of neighboring, cells [11–15]. For a square lattice of L by L cells, we randomly throw sand grains, one at a time, onto the grid. In the original Bak-Tang-Wiesenfeld (BTW) sandpile model, once the total amount of accumulated sand on a single cell reaches the threshold amount of four, the sand will either be redistributed to four adjacent cells (the nearest neighbors) or fall off the edge of the grid. Our modified LRCS model differs from the BTW model in terms of releasing toppled grains to the four nearest-neighboring cells. The modified rule of random internal connections is similar to the implementation of Watts and Strogatz [16]. For any particular cell, when the accumulated grains exceed the threshold and redistribution occurs, one of the original nearest-neighbor connections has a long-range connective probability P_c of being redirected to a randomly chosen, distant cell. The original connection is replaced by a randomly chosen mesh that may be far from the toppling cell. For a scheme of the distribution process of the LRCS model, please refer to our previous studies [12–15].

We have additionally assumed that P_c depends strongly on any topographic changes induced by the last event, which is defined as $P_c(t+1) = [\Delta Z(t)/\alpha L^2]^3$. $\Delta Z(t)$ and L^2 are topographic changes due to the latest event and the system size, respectively. The coefficient α functions as a normalization constant, which makes the value of the connective probability P_c range between 0 and 1. The LRCS model after a large avalanche can thus induce a high probability of long-range connections, which is motivated by the fact that a more active earthquake fault system will have a higher likelihood of establishing long-range connections due to many factors, such as fault activity, the change in pore fluid pressure, or the dynamic triggering of seismic waves. For example, a larger earthquake generates a more radiated energy that is carried by seismic waves; therefore, it is more capable of dynamically triggering remote tremors that are far from the main shock. In those remotely triggered cases, stress perturbation due to seismic waves is considered to be the immediate cause of triggered events [17,18]. Figure 1 shows the power-law frequency-size distributions of avalanches for the original BTW sandpile and our modified LRCS models [12]. Overall, both models have similar power-law frequency-size distributions and mimic the Gutenberg-Richter law of real-life earthquakes. By using a self-adapted probability threshold P_c for a remote connection, the self-adapted LRCS model demonstrates a state of intermittent criticality [19–21] in which the sandpile intermittently approaches and retreats from the critical state (fig. 2). In the LRCS model with a self-adapted P_c, the dynamic variable for the spatially averaged amount of grains on board,

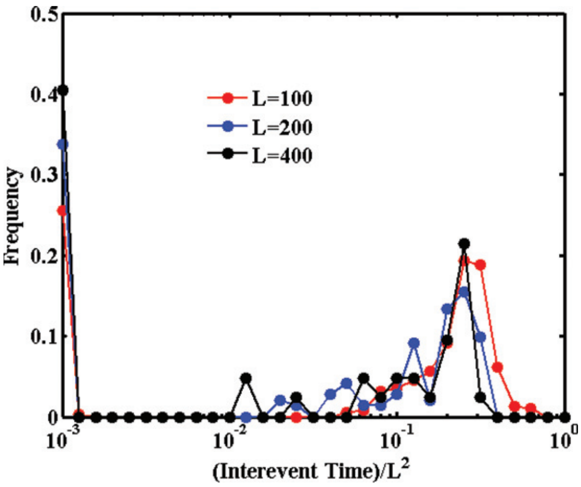

Fig. 3: (Colour on-line) The probability distribution of the *scaled* intervening intervals for large avalanche events in the LRCS models with $L = 100$, 200, and 400.

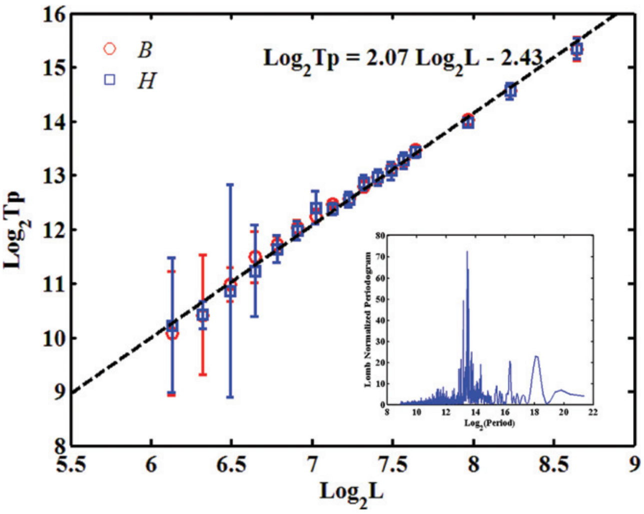

Fig. 4: (Colour on-line) The correlation between sandpile sizes L and predominant periods T_{p} obtained from time series of B's and H's. The error bars show the variance of T_{p} obtained from ten realizations of the LRCS models for each L from 70 through 400. The insert shows an example of the Lomb spectrum for a time series of the B values calculated from one LRCS model with $L = 200$.

$\langle Z\rangle(t) = (\sum_{i=1}^{L^2} Z_i(t))/L^2$ (blue line in fig. 2), often favors smaller values by large events (black bars in fig. 2). The large fluctuation in $\langle Z\rangle(t)$ is an important feature mimicking the intermittent criticality [19–23]. Large avalanches are then followed by a period of quiescence and a new approach toward the critical state [12–14]. This process is similar to the dynamic process of the earthquake fault system, which repeats by reloading elastic strain energy and rebuilding correlation lengths towards criticality and the next large event [22–24]. For more details about the LRCS model, we refer readers to our previous papers [12–15].

Quasi-periodicity of large avalanches in the LRCS model. – Figure 3 shows the frequency distribution of the recurrence intervals between two successive large avalanche events in the LRCS models with various system sizes, $L = 100$, 200 and 400. A large avalanche here is defined as an event with the toppled size s exceeding $0.99\,L^2$, which indicates an event that spreads throughout most of the sandpile system. The recurrence interval of the event has been rescaled by the system area L^2 so that the distributions with various L's can collapse together. An important feature in fig. 3 is the approximately bimodal distribution of recurrence intervals. Strikingly, we find a substantial mode of those rescaled intervals at approximately 0.25, which indicates that the large events in the LRCS models with various L's are quasi-periodically distributed throughout many realizations. Another mode for the small values (*i.e.*, less than 0.05) of the rescaled intervals is caused by event clustering, which may be associated with the foreshock-mainshock-aftershock sequences.

We conjecture that the dynamics of the LRCS model shows weakly periodic behavior. We have previously shown that the power-law exponent B of the frequency-size distribution for the LRCS model exhibits large fluctuation related to large avalanche events [13,14,25]. Also the Hurst exponent H of the avalanche sizes is consistent with these results. Figure 2 shows an example of the LRCS model with $L = 150$, demonstrating the temporal variations in the B (red line) and H (green line) values. Gradually approaching a large event, the B value decreases, and the H value in the increases. If large avalanches are quasi-periodic, then the fluctuations in the B and H values will have periodic behavior.

For calculating the B and H values, we collected a fixed number of events, thus generating irregularly spaced time series. An effective approach to examine the periodicity of the dynamical fluctuation in the LRCS model invokes Lomb periodograms. Fourier spectrum is a standard technique to detect the periodicity of a time series with equally spaced samples, while the Lomb periodogram is used for cases with irregularly spaced time series data. We used Lomb periodograms to determine the periodicities for several time series of B's and H's. A predominant period T_{p} in the fluctuations of B's and H's can then be detected as the inverse of the frequency for the peak with the highest power spectral value. We found strong evidence that further supports the quasi-periodic dynamics of the LRCS model in the Lomb spectra for time series of the B and H values generated by the LRCS models. An example with $L = 200$ can be found in the insert in fig. 4. Most importantly, fig. 4 shows a strong correlation between T_{p} and L. The error bars show the variance of T_{p} obtained from ten realizations, each with 1 million iterations of sand throwing, of the LRCS models with L from 70 through 400. It is reasonable that large avalanches in a larger-sized LRCS model have, on average, longer recurrence times. However, such recurrence behavior is not purely periodic (fig. 2). The strong dependence between T_{p} and L is exhibited in the LRCS models as the scaling of $T_p \propto L^{2.07}$, which is consistent with the rescaled intervals in fig. 3.

The described conjecture can be theoretically formulated and proven to be true. Let the number of toppled sites during an avalanche be denoted as s and the probability distribution of s on a L^2-sized lattice be denoted as $P(s, L)$. If $P(s, L)$ follows the finite-size scaling, then it could be written as

$$P(s) = s^{-\tau} G(s/L^D), \tag{1}$$

where τ and D are two scaling exponents and G is a finite-size scaling function. If we call an avalanche with $s \geq aL^2$ a large event, then the probability of a large event will be

$$\Delta P = \int_{aL^2}^{L^2} P(s)\mathrm{d}s.$$

Then

$$\Delta P = \int_{aL^2}^{L^2} s^{-\tau} G(s/L^D)\mathrm{d}s = L^{D(1-\tau)} \int_{aL^{2-D}}^{L^{2-D}} (s/L^D)^{-\tau} G(s/L^D)\mathrm{d}(s/L^D) \cong L^{D(1-\tau)} \int_a^1 u^{-\tau} G(u)\mathrm{d}u, \tag{2}$$

where $\int_a^1 u^{-\tau} G(u)\mathrm{d}u$ is a constant and the approximation could be reasonable for a D of ~ 2.

For L values from 100 through 400, the LRCS simulation generates approximately 40000 events during 100000 iterations of sand-throwing, which is independent of L. The inverse of ΔP is therefore easily converted to the average waiting time (iteration) between two large avalanches. Remember that larger events are often clustered for the LRCS model (fig. 2), mimicking the foreshock-mainshock-aftershock sequence. Imagine that there are N_T avalanches in a cycle of large events and that among these, N_a events larger than aL^2 are clustered. Therefore, $\Delta P = N_a/N_T$. We have noted that the number of clustered large events scales with L. The recurrence period of large events can be then rewritten as

$$T_p = t \times N_T = t \times N_a/\Delta P \propto L^m/L^{D(1-\tau)} = L^{D(\tau-1)+m}, \tag{3}$$

where t, roughly equal to 2.5 in the LRCS models, is the average iteration number for one avalanche and m is the scaling exponent between N_a and L. The values of D and τ can be obtained from the means of the moment analysis for the LRCS models and are 2.02 and 1.79, respectively. Figure 4 thus suggests that m nearly approaches 0.48, which is consistent with the distribution shown is fig. 3. Note that the number of clustered large events for $L = 400$ is nearly twofold larger than the *mainshock* number and is comparable for $L = 100$. Nevertheless, we have a theoretical foundation for the L dependence of the recurrence intervals of large avalanches in the LRCS models. The recurrence interval can be related to the size of LRCS system which is then associated with and the magnitude of the so-called characteristic event.

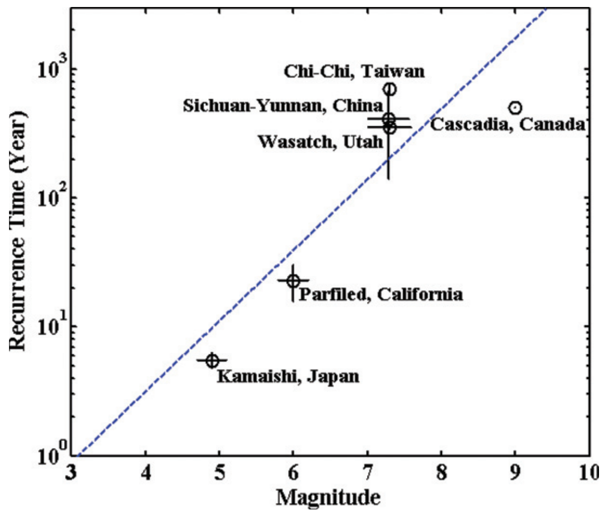

Fig. 5: (Colour on-line) The correlation between recurrence times and magnitudes for natural earthquakes [1,24–28]. The trenching data of faults are used primarily to identify the paleoevents, particularly for those cases with magnitudes larger than 7.

Discussion and conclusions. – Could we find a similar size/magnitude dependence for the recurrence times of natural earthquake fault systems? In addition to the Parkfield, California earthquake sequence, which has a magnitude of $\sim$ 6.0 and a recurrence interval of $\sim$ 22 years [1], Uchida et al. found that earthquakes with magnitudes of $\sim$ 4.9 that occurred on an interplate asperity near Kamaishi, Japan have a strong periodicity of $\sim$ 5.5 years [26]. By using the paleoearthquakes observed in the trenching data for the magnitude 7.3 Chi-Chi, Taiwan earthquake, Chen *et al.* [27] suggested an average recurrence interval of $\sim$ 700 years. The recurrence time for the magnitude 9.0 Cascadia, Canada earthquake is approximately 500 years [28]. Other cases include the Wasatch fault zone in Utah [29] and the Sichuan-Yunnan fault zone in China [30]. The average recurrence times of the Wasatch and Sichuan-Yunnan fault zones are $\sim$ 340 and $\sim$ 410 years, respectively. The events in both areas have comparable magnitudes of $\sim$ 7.2. For natural earthquake fault systems, paleoearthquakes observed in the trenching data are the only way to calculate the recurrence periods for large characteristic events, but this approach gives great uncertainty. Limited instrumental data are available due to the (fortunate) lack of repeated experience with these catastrophic events. However, for all the above mentioned cases, fig. 5 does show a satisfactory correlation between recurrence times and magnitudes, *i.e.*, $T_\mathrm{p} \propto M^{0.549}$. This power-law relationship with a scaling exponent of 0.549 differs from the value we determined with the LRCS model. This exponent must be transformed to obtain the exact scaling between the recurrence times and the linear dimension of natural earthquake fault system, which could be complicated because of fault geometry. This issue is beyond the scope of this study and will be examined in future work.

Our motivation of the long-range connection in the LRCS model adopted the random wiring of the small-world network. The difference is that the probability of long-range connection in the small-world network system is purely random. Instead, the probability of long-range connection in our LRCS model depends on the size of the previous event. As shown in this study, the LRCS model represents a system having a quasi-periodical behavior. In fig. 2 the variation of B-values, H-values and the dynamic variable for the average topographic height of the board are showing the quasi-periodicity with large events. We suggest that the quasi-periodical behaviour should be caused by the size dependence of the long-range connection probability. For large events which can occur long before or after the predominant periodicity. The Parkfield earthquake in 2004 is likely a quasi-periodic case. The 2004 Parkfield event could be the expected characteristic earthquake, and its late occurrence is not surprising in view of the quasi-periodical LRCS system. The LRCS model may be a new physical model of earthquakes and differs from time-dependent and slip-dependent models [31,32]. In the LRCS model, the gross cumulative energy before a large event can be changed as can the energy released during a large event ($\langle Z \rangle(t)$ in fig. 2). Such variance, together with the long-range connection effect, may be related to biases in the recurrence intervals of large characteristic avalanches. The determination of the size/magnitude dependence of the recurrence times for large earthquakes is important. A larger earthquake needs longer preparation time to re-accumulate energy before its reoccurrence. From the point of view of risk prognosis, the occurrence probability through time of characteristic events is an important issue to study. We are still far from the accurate prediction of earthquakes, although most of the *rescaled* recurrence intervals of large avalanches are ~ 0.25 (fig. 3). For now, to measure the probability of characteristic earthquakes at any given time is still a challenge. The uncertainties of paleoseismic dating, and the quantification of rupture dimension, fault segmentation and magnitude are important effects on the prediction of earthquakes. Many studies about statistical analysis and statistical models of earthquake forecasting are still ongoing (*e.g.*, accelerating moment release (AMR), characteristic earthquakes, variation in b-value, activation model, and Pattern Informatics (PI) index), those studies have seen significant progress in the last ten years [33–35].

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